SDT non-linear transient and HBM toolbox 0.3

User guide

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Theory and reference

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1.1 Kinematics of non-linear systems

1.1.1 Observation of strains, stresses, application of forces

One is interested in solving equations of the general form

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} + [K]\{q(t)\} = F_{NL}(q, \dot{q}, q_{NL}, \omega, t) + F_{ext}(\omega, t) \tag{1.1}$$

with q the finite element DOFs, M,C,K the mass, viscous damping, and stiffness matrices respectively, F_{ext} the external forces and F_{NL} the non-linear forces internal to the system which may depend on the FEM motion described by its DOFs q and their velocities $\dot{q}$ as well as possibly internal states of the non-linear elements q_{NL}. When internal states are necessary (friction, plasticity, ...), additional equations are provided to model their evolution.
Estimation of the non-linear forces can,, in a very general fashion, be decomposed in three steps: observation of strains, evaluation of constitutive law at a material point to compute stresses, application of stresses on the model as detailed below.

- **Observation of non-linear strains** is the first step

  $$\{\epsilon(t)\} = [c]\{q(t)\} \tag{1.2}$$

  where strains may represent quantities dependent on the model displacement (relative motion, mechanical strain, ...), but also possibly internal states of the non-linearity if those are included in the definition of q. For HBM solutions q_{NL} will be assumed to be part of q but for transient simulations this may not be optimal.

  Strategies for the construction of the observation matrix c will be discussed for point to point connection in section 1.1.2 , surfaces in section 1.1.3 , volumes in section 1.1.4 .

  In the time domain, generalized strains ϵ (noted `.unl` in the code) are obtained by computing

  $$u_{nl} = [c]\{q\} + u_{nl0} \tag{1.3}$$

  In the implementation, the strain vector may have N_E components e_i, and strains at N_G material points (Gauss or physical points) may be stored as a single vector to allow vectorization of non-linearities of a given kind, thus leading to $e_{i,gk}$ components.

  For HBM computations N_T times will be evaluated and stored as columns. The internal storage in field `NL.unl` is thus of the form

  $$\{u_{nl}\}_{(N_E\times N_G)\times N_T} = \{\epsilon\}_{\text{gauss repeat}\times\text{time}} = \begin{bmatrix} e_{1,g1}(t_1) & e_{1,g1}(t_2) & \dots \\ e_{2,g1}(t_1) & \dots & \\ \vdots & & \\ e_{1,g2}(t_1) & \dots & \\ \vdots & & \end{bmatrix} \tag{1.4}$$

In a similar fashion strain rates are obtained using

$$v_{nl} = [c]\left\{q'\right\} + v_{nl0} \tag{1.5}$$

and stored in field `NL.vnl`.

- From the observed strains, a constitutive law is used to **estimate stresses** (or generalized forces), as will be discussed in section 1.2 from data present in the `NL.Fu` field,

$$\{s_{nl}(t)\} = F\left(\{\epsilon(t)\}, \{\dot{\epsilon}(t)\}\right) \tag{1.6}$$

  The generic representation of non-linearities should verify classical assumptions on objectivity and on the validity of constitutive relations, which is compatible with the idea that generalized strains are defined at a material point.

  The definition of a non-linear constitutive relation giving a definition of generalized stresses s_{nl} which has the same $(N_E \times N_C) \times N_T$ size as u_{nl} and is thus stored `NL.unl` field to allow overwrite. This allows memory optimization even though the output result is a force and not a strain). When internal states are present, the strain field `NL.unl` may not be efficiently used to store stresses, it is then possible to store stresses in field `NL.snl` of size $(N_S \times N_G) \times N_T)$ with N_S the number of stress components.

- Finally stresses can be applied on the model to **obtain the model forces** in the discretized model associated with DOFs q. That is

$$\{-F_{nl}\} = [b]\{s_{nl}\} \tag{1.7}$$

  where the field `NL.snl` may be defined or, in cases with no internal states, it is possible to optimize memory by storing s_{nl} in `NL.unl`.

In the proposed framework,

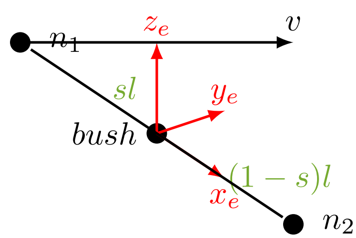
- internal states of a non-linearity are expected be included within the generalized strains. For a time computation `unl` will thus be of size $(N_E + N_{int}) \times N_g$ $(\times N_T$ in the case of HBM computations). See section 1.2.6 for more details.
- It is generally useful to store current $u_{nl}(t_n)$, initial $u_{nl0}$and previous $u_{nl}(t_{n-1})$ values as consecutive blocks in `NL.unl(:,:,1),NL.unl(:,:,2),NL.unl(:,:,3)`.
- In many instances the observation c and command b matrices can be considered constant, but for contacts with large relative displacements they may need to be considered as operators depending on the model state and thus modified dynamically using C code.

1.1.2 Generalized non-linear springs

The simplest example is the observation of unaxial springs oriented along vector $\{d\}$ where the generalized strain is the relative displacement

$$dq = \begin{bmatrix} -\{d\}^T & \{d\}^T \end{bmatrix} \left\{ \begin{array}{c} u_1 \\ v_1 \\ w_1 \\ u_2 \\ v_2 \\ w_2 \end{array} \right\} \tag{1.8}$$

The non-linear implementation of the OpenFEM `cbush` element provides 6 generalized strains at a given location corresponding to relative translations and rotations. For `EDID==-1` this is with respect to a local basis $B = [x_e \; y_e \; z_e]$.



$$\left\{ \begin{array}{c} du \\ dv \\ dw \\ ru \\ rv \\ rw \end{array} \right\}_{NE\times 1} = \begin{bmatrix} -B^T & 0 & B^T & 0 \\ 0 & -B^T & 0 & B^T \end{bmatrix} \left\{ \begin{array}{c} u_1 \\ v_1 \\ w_1 \\ ru_1 \\ rv_1 \\ rw_1 \\ u_2 \\ v_2 \\ w_2 \\ ru_2 \\ rv_2 \\ rw_2 \end{array} \right\} \tag{1.9}$$

Initialisation of non-linear behavior in a `cbush` element group is performed when the NLdata property contains fields

- `type='nl_inout'` to let `hbm_solve InitHBM` build the needed observation matrix

- Fu='@UserFun' references a user function computing the non-linear force with the prototype call described in section 1.2.4
- .isens can be used to apply non-linearity in some directions only.

A sample problem with cbush can be found in section 2.3.3 .
When considering non small angular rotations of the two nodes, the orientation basis in (1.9) cannot be considered considered constant. It is thus necessary to express motion of each part at the bushing node using large rotations

$$\{x_{iB}(t)\} = \{x_i(t)\} + [R_i(t)]\{x_{iB}(0) - x_i(0)\} \tag{1.10}$$

In non-linearities that implement such large rotation, SDT expects that the bushing frame is associated with the first node and that motion of the bushing node associated with the second. Thus the relative translation in first node frame is given by

$$[R_1(t)]^T\{x_{2B} - x_{1B}\} = [R_1(t)]^T\{x_2(t) - x_1(t) - x_1(0) + x_{1B}(0)\} + [R_1(t)]^T[R_2(t)]\{x_{2B}(0) - x_2(0)\} \tag{1.11}$$

For rotational springs at the bushing, there is no obvious large rotation formulation. The current implementation uses the difference of angles even though this does not always make sense.

1.1.3 Generalized non-linear surfaces

Zero thickness elements which provide 3 generalized strains which correspond at each gauss point to the relative motion of two surfaces in the normal and two tangential directions are implemented in p_zt. When an NLdata field is defined a vectorized non-linearity is initialized.
Contact elements, implemented in p_contact have an objective similar to that of zero thickness element but allow the handling of non-conform meshes. Subtype 1 does not account for large displacement so that matching can be performed once and kept constant during time. Subtype 2 allows for relative motion of two bodies (wheel moving on a rail for example) and currently only works for matching to triangular surfaces.

1.1.4 Non-linear volumes

SDT implements observation of strains in configurations where no geometry updating is needed using StressCut calls. This strategy is compatible with all physics (acoustics, piezo-electricity, ...) and can also be used for shells (to observe membrane strains and curvature) and beams (to observe axial elongation, shear, torsion and curvature).
For volumes, the choice of *strain* is obtained using the p_solid Isop parameters. For explicit mechanics in large deformation, the displacement gradient is used and obtained using Isop=100.

For generic implementation of *multi-physic non-linear volumes*, a generalized strain giving the displacement and its gradient for each field. For a field with 3 components at N nodes

$$\{e_{MP}\}_{12} = \begin{Bmatrix} u_1 \\ u_{1,j} \\ u_2 \\ u_{2,j} \\ \vdots \end{Bmatrix}_{12} = \begin{bmatrix} N & 0 & 0 \\ N,x & 0 & 0 \\ N,y & 0 & 0 \\ N,z & 0 & 0 \\ 0 & N & 0 \\ 0 & N,x & 0 \\ 0 & N,y & 0 \\ 0 & N,z & 0 \\ 0 & 0 & N \\ 0 & 0 & N,x \\ 0 & 0 & N,y \\ 0 & 0 & N,z \end{bmatrix} \begin{Bmatrix} u_1(n1) \\ u_2(n1) \\ u_3(n1) \\ \vdots \end{Bmatrix}_{3N} \tag{1.12}$$
$$= [B_{MP}(r,s,t)]_{12\times 3N}\,\{u_{in}\}_{3N}$$

As a an example, the standard linearized mechanical strain

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \gamma_{yz} \\ \gamma_{zx} \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} N,x & 0 & 0 \\ 0 & N,y & 0 \\ 0 & 0 & N,z \\ 0 & N,z & N,y \\ N,z & 0 & N,x \\ N,y & N,x & 0 \end{bmatrix} \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = [B_{Ep}]_{6\times 3N}\,\{u_{in}\}_{3N} \tag{1.13}$$

can be related to the generic multi-physic strain using a transformation matrix T_{Ep}

$$[B_{Ep}]_{6\times 3N} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} [B_{MP}]_{12\times 3N} = [T_{Ep}]\,[B_{MP}] \tag{1.14}$$

The usual mechanical stiffness can thus be reformulated using B_{MP}

$$K_e = \int_\Omega [B_{Ep}]^T\,[D]\,[B_{Ep}]\,d\Omega = \int_\Omega [B_{MP}]^T\,\left[T_{Ep}^T D T_{Ep}\right]\,[B_{MP}]\,d\Omega \tag{1.15}$$

A generic non-linear multiphysic constitutive law is a function that will receive a `NL` structure with fields

- `.unl` a series of generic strain e_{mp} at each Gauss points thus a matrix with `4*Nf*Ng` rows (4 components per field, repeated for each gauss point), and possibly multiple columns for different times.
- `.vnl` a series of strain velocities (time derivative of strain)
- `.nodeG` matrix of size `Nfield * Ng` containing interpolated fields (typically positions $x, y, z, un1$ but possibly also `v1x,v1y,v1z` for first orientation axis), `.nodeEt` is an int32 vector coding the name of each field.
- `.MatType` list of matrix types for which the tangent constitutive law is given.
- `.constit` contains the constant parameters read from `model.pl`.

From this information the function should return a structure with field

- `.unl` generic stresses a matrix with `4*Nf*Ng` rows and `Nt` columns. The name should really be `.fnl` but the same field name `.unl` is used to allow memory reuse.
- `.MatType` propagated version of input field.
- `.DD` a matrix of size `ones(4*Nf,4*Nf,Ng,length(NL.MatType))` giving the tangent constitutive law for each Gauss point and each desired matrix type. This corresponds to the $T_{Ep}^T D T_{Ep}$ in (1.15).

1.1.5 Kinematic reduction, observation and hyperreduction

Once a kinematic reduction defined, where $\{q\} = [T]\{q_R\}$ is defined, non-linear observation and command matrices are simply reduced using $c_R = c * T$ and $b_R = T^T b$.
For resultant observations, the strategy is however more complex. In a frequency domain model, the resultant is associated with $Z_I(p,s)$ the dynamic stiffness restricted to an interface

$$R_I == [c_I]\,[Z_I(p,s)]\,\{q(\omega)\} = [c_I]\left[Ms^2 + \sum \alpha_p M_p\right]\{q(\omega)\} \tag{1.16}$$

so that the coefficients of the observation depends on frequencies and parameters involved in $Z_I(p,s)$. In SDT, such linear combinations as described in section **??** . Taking the case of a bushing where $[Z_I] = E_(\omega, a)\,[K_u]$, the resultant observer is simply given by $[c_I]\,[Z_I] = E_(\omega, a)\,[c_I K_u]$.
When the observer is built around an operating condition where large displacement has occurred, the observation matrix may depend on the current position $c(x)$. For example, the elongation of a large rotation rod is given by $dl(t) = \sqrt{\{u_2 - u_1\}^T \{u_2 - u_1\}} - l_0$, so that the linearized observation is given by

$$dl(t) = \{e_i(u_0)\}^T \{u_{i2} - u_{i1}\} = \frac{1}{\|u_2(0) - u_1(0)\|} \{u_2(0) - u_2(0)\}^T \{u_{i2} - u_{i1}\} \tag{1.17}$$

For the case of resultant observation in enforced displacement problems, $R_I(t) = [T_I]^T F(t)$, the reduced equations of motion give an estimate of $F_R(t)$ and not F_T, thus T_{IR}^T should be used when observing.
Hyperreduction is a strategy where kinematic reduction is combined with selection of a subset of non-linear integration points based on criteria on the work of reduced model. Thus from a set E of gauss points where each non-linear strain is described by $u_{nl}^g = [c^g]\{q\}$, one restricts to a subset E_{HR}. Needs more details.

1.1.6 Manual definition of input and output (deprecated)

While the general approach is to associate non-linearities with strains in elements as described in the following sections, it is possible to define a `pro.NLdata` structure giving

- `.b,.c` : manual command and observation definition. If the `NLdata.DOF` field is defined, placement of the observation matrix in done during the model assembly phase assuming `.DOF` correspond to mdof (projection with the `Case.T` matrix, $c * T$,is done). If one want to define NL on active DOF, one will provide `.adof` field (rather than `.DOF` field): then c is assumed to be defined on active dof, and a simple placeindof is done. If there is no `NLdata.DOF` field, `.c` and `.b` matrices are assumed to be on `mdof`, and projected using `Case.T`. For storage during solves, see `.b`.
- `.Sens,.Load` Alternate form for `.b, .c` giving command and observation matrices as `.Sens` cell array of the form `{SensType,SensData}` where SensType is a string defining the sensor type and SensData a matrix with the sensor data (see `sdtweb sensor`). `.Load` data structure defining the command as a load (with `.DOF` and `.def` fields).

1.2 Non-linear constitutive laws

While implementation of non-linear kinematics is fairly generic, users typically want to implement their own constitutive laws relating stresses to strains and possibly their history.

1.2.1 Laws with no internal states, principles

Constitutive laws where the stress only depends on strain and strain rate are the simplest. These laws exploit the framework provided by `nl_inout` for various element types.
The only thing that needs to be implemented is a `.Fu` function

$$\{s_{nl}(t)\} = F(u_{nl}, v_{nl}) \tag{1.18}$$

During time/frequency evaluations it is essential that such evaluations be very fast, this has an impact on implementation and different strategies are implemented

- tabular section 1.2.2 .
- dedicated user function section 1.2.4 (deprecated).
- anonymous function defined with parameters section 1.2.5 (deprecated).

1.2.2 Tabular interpolation

```
sdtweb('_eval','d_fetime.m#BumpStop')
opt=d_fetime('timeopt dt=1e-4 tend=.1');opt.Method='Back';
model=fe_time(opt,mdl)
```

Multiple forms are supported. Currently a cell array of

- tabulated Fu, ie 1st column is the observed strain (relative displacement), 2nd column F_{u1} is the corresponding stress (load). A third column F_{u2} is interpreted as a coefficient applied to computed non linear load associated to velocity (Fv) (it is used for example in bumpstops to take in account a damping with offset on the displacement). Thus $s_{nl} = F_{u1}(u_{nl}) + F_{u2}(u_{nl})F_v(v_{nl})$.

  When each non-linearity has 3 strains, this is interpreted as contact. You are expected to have non-linear kinematics giving strains as normal component followed by two tangent directions. Thus $s_1 = F_{\text{normal}} = F_{u1}(u_1)$ and $s_2 = F_{\text{Tangent1}} = F_{u2}(u_1)F_v(v_2)$ $s_3 = F_{\text{Tangent2}} = F_{u2}(u_1)F_v(v_3)$.

  When storing constitutive laws in tabular form, it is desirable to follow the SDT `curve` format giving the the variable names on which the function depends in the `.Xlab` fields, abscissa in the `.X` and values in the `.Y` field.

- numeric curve ID for a curve in the stack.
- string defining a predefined law, listed using `nl_spring('guilist')`.

For Jacobian computations by `nl_spring NLJacobianUpdate`. One uses
xxx
Supported laws are

- `BumpStop dp` *dp* `kp` *kp* `cp` *cp* `dm` *dm* `km` *km* `cm` *cm* (a bumpstop example can be found in `sdtweb t_nlspring('BumpStop')`. *dp* is the upper gap of the bumpstop, *kp* the upper stiffness, and *cp* the upper damping, then *dm* *km* and *cm* the same for the lower gap. This bump stop law (as friction law) is built in `nl_spring Tab` which is the historical implementation : there is a known approximation, the stiffness is applied from $du = dp * 1.001$ to $du = 1$ (so that the slope is not exactly kp), and damping from du=dp. The same for lower gap.

- `Friction f` *f* `c0` *c0*, where *f* is the friction load, and *c0* is a damping coefficient applied on the transition around 0 velocity (*c0* is typically important). Dry frictions are known to be responsible for convergence problems.
- `gapCylI` inner cylinder within an external sleeve.
- `ctcFric` penalized contact and friction implementation

1.2.3 User callback in `nl_inout`, MexCb field

The current high performance developments are focusing on vectorized user implementation of non-linearities integrated in the `chandle` `nl_inout` implementation. In the data structure used during time integration (see section 1.5.3), the `.MexCb` field is used to provide data. It is a cell array giving `{@fun,RO}`.
For optimized operation limiting field name checks, the option structure `RO` is assumed to have ordered fields

- `.unlg` at a single Gauss point,
- `.opt` copy of `NL.opt`,
- `.jg` int32 vector allowing passing of current Gauss point index,
- `.vnlg` optional copy of velocities at Gauss point or empty.

xxx

1.2.4 Dedicated user function (deprecated)

The easiest conceptual way to define a non-linearity is to use your own function. For example, if you have defined the function

```
function NL=resCubic(NL,fc,model,u,v,a,opt,Case,RO)
NL.unl=-.01*NL.unl.^3  ... % Cubic stiffness on relative motion
     + -.02*NL.vnl;        % Linear viscous damping on relative velocity
```

You can specify that this function should be used to compute a law with no internal states using

```
model=d_fetime('TestbeamNL');
model=nl_spring('SetPro proid100 Fu="@resCubic"',model);
% verify that Fu was defined in NLdata
NL=stack_get(model,'','NL','get');NL.NLdata.Fu
% The same with a subfunction in d_hbm
model=nl_spring('SetPro proid100 Fu="d_hbm(''@resCubic'')"',model);
NL=stack_get(model,'','NL','get');NL.NLdata.Fu
```

Note that in instances of deployed MATLAB generated with the MATLAB compiler, all custom functions must be defined a priori. And only anonymous functions may be created.
The `NLdata` entry is kept as the one defined by the `nl_inout db` call. Entry `NLdata.Fu` must then be replaced by the handle to the dedicated function in the non-linear property

1.2.5 .anonymous field for definition (deprecated)

To allow parameter edition, the base mechanism to automatically create an `.Fu` as MATLAB anonymous function handle is to use the following `NLdata` fields

- `.anonymous` a string which upon model initialization in `nl_spring('Init')` will be transformed to an anonymous function of the form

  `Fu=@(NL,~,~,~,~,~,opt,~)AnonymousString`

  The `@(...)` part can be included in the string if it differs from the default.

  Parameters are fields of the `NL` non-linear structure (in the example below `NL.par1` will be defined). The initialization of these parameters is performed using the `NLdata.Param` field described below.

  In frequency domain solvers, the current frequency (rad/s) is accessible in `opt.w`.

- `.csv` a string to define parameters in the anonymous function. This string declares parameters using `cingui ParamEdit` format. This declares the parameters to be used, with a default value, their type and a possible brief explanation, as illustrated below.

- `.Param` The current parameters. `.Param` can be

  - a string defining the parameters declared in the `.csv` by `par1=val1 par2=val2 ...`
  - a structure with fields corresponding exactly to the declared parameters `struct('par1', val1, 'par2', val2)`.

  Any omitted parameter will be set to its default declared in `.csv`. Lack of default values would then results in an error at the function execution.

- `.tex` a string providing a `tex` format of the formula used in `.anonymous`.

Use of inline functions. One can directly use the existing framework with a customized call based on the concept of *anonymous function handles* in MATLAB.

```
d_hbm('TestDuffing2Dof-an')
% completes the defintion
NLdata=struct('csv','par1(1#%g#"value of parameter 1")',...
'Param','par1=val',...
```

```
'tex','p_1 u_{NL}'
'anonymous','-NL.par1*NL.unl');
model=feutil('setpro 2001',model,'NLdata',NLdata);
```

1.2.6 Laws with internal states

Classical rheologic model exploit internal states to account for hysteresis phenomena. *E.g.* The elasto-visco-plastic behavior is shown in figure 1.1 In this case one internal state is required to

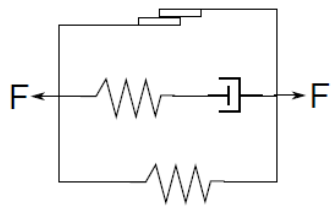


Figure 1.1: Elastic-visco-plastic model

represent the relative force between both extremities, to keep track of the relative displacement between the middle spring and middle dashpot.
Very complex models can be associated to this kind of representation, where one will implement internal dynamics associated to a strain history. The general formulation of such system is given as

$$f_{NL} = \mathcal{F}\left(\{q\},\{\dot{q}\},\{z\},\{\dot{z}\},t\right) \tag{1.19}$$

and the internal states evolution equation functional

$$\{\dot{z}\} = \mathcal{G}\left(\{q\},\{\dot{q}\},\{z\},t\right) \tag{1.20}$$

where $\{z\}$ is a vector of internal states, $\{q\}$ the displacement vector and $\dot{x}$ represent the time derivatives.
Equations (1.19) and (1.20) represent the class of so-called state space models (for example in the 80's for geophysics applications, in particular by Rice and Ruina [**?**]).
Internal state representation can be based on the need for an efficient implementation and on the fact that the first order dynamics laws defined by equations (1.19) and (1.20) do not comply with a second order based resolution framework (the internal states acceleration is seldom defined).
Equation (1.20) can thus be resolved separately, possibly with a sub-integration scheme and an adequate interpolation of the external states.

In the non-linearity framework internal states are stored in the continuity of the generalized strains per Gauss points in field `.unl`. A single non-linearity only handles a single topology and a single internal model, so that each Gauss point has the same number of strain observations and internal states. The field `.unl` is then of size $((N_E + N_I) \times N_G) \times N_T$. The internal storage in field `NL.unl` is thus of the form

$$\{u_{nl}\}_{((N_E+N_I)\times N_G)\times N_T} = \{\epsilon\}_{\text{gauss repeat}\times\text{time}} = \begin{bmatrix} e_{1,g1}(t_1) & e_{1,g1}(t_2) & \dots \\ e_{2,g1}(t_1) & \dots & \\ \vdots & & \\ e_{1,g2}(t_1) & \dots & \\ \vdots & & \\ e_{1,i1}(t_1) & & \\ \vdots & & \end{bmatrix} \tag{1.21}$$

This formalism keeps vectorization capability per instant. The internal rate states are stored in the same manner.
Strain history is stored in the t̂hird dimension `.unl(:,:,jh)`. In general for time integration, one will use

- `.unl(:,:,1)` to store the current strains of the form (1.3)
- `.unl(:,:,2)` for the initial strains u_{nl0}
- `.unl(:,:,3)` to store $u_{nl,t-1}$ the strains at the previous step

When performing a **residual** call, it is efficient to combine time stepping (1.20) and state update. The `StoreType` parameters controls what the `StoreState` C function does.

- `StoreType=01` single step computation of residual and update of internal states in u_{nl}. The memory buffer of `.unl(:,:,1)` is first filled using (1.3) but assuming that internal states have not changed. The residual function computes a step replacing the data in `.unl(:,:,1)`. `StoreState` (done at end of `Fu`) copies `.snl` to `.FNL` . Copy of internal states to `unl(:,:,2)` can be done by controlled by the `Fu` (this is in particular is done in the `StoreState` C++ function when ûpUnl2=RO.jg[2]=2 is used).
- `StoreType=02` stores internal states in `.FNL` rather than `.snl`.
- `StoreType=03` same as 01 but `StoreState` stores the full `.snl` vector followed by the full `.unl(:,:,1)`. Beware that, this can be quite memory intensive.
- `StoreType=04` used for enforced motion. Modifies the displacement and velocity buffers.

- StoreType=1.. if the hundreds digits is set to 1, .vnl is assumed not used and thus the associated buffer is emptied. xxx need discuss xxx

When .iopt(1)=FInd is positive, .snl is copied as a consecutive vector to model.FNL assuming the field exists. When .iopt(2)=iu is positive, internal states are copied consecutively to the displacement vector u with offset iu. This is done for Ng=.iopt(5) gauss points while skipping Nstrain=.iopt(3) components and copying Nistate=.iopt(4).
only the previous state is needed to integrate internal state evolution, as offset u_{nl0} is already stored in the third dimension, $u_{nl,t-1}$ is found in the third index.
DOF representation of internal states

The number of internal states depends on the model and thus must be declared in the non-linearity. One must then declare in NLdata the fields .adofi and field .MatTyp to obtain a proper initialization of .unl field and associated observations in nl_inout.

- Internal states are defined independently for each observation line used in the non-linearity. *e.g.* for a cbush six directions are available relative to the 3 translations and 3 rotations that can be observed. .adofi is then a line cell array of length the number of observations (6 here). Each cell defines a number of internal states associated to the corresponding observation index by providing a column vector with as many lines as internal states used each containing the DOF extension .99. The cell is left empty if no internal state is declared for a particular direction.

  Thus NLdata.adofi={[];[];[];[];[];.99} will add an internal state to the 6^{th} observation of the non-linearity.

  .99 adds the internal state as an additional DOF, while -.99 uses an FNL internal state.

- Field .adofi must be either a one column vector/cell (Gauss point wise replication is supported) or should feature as many columns as expected Gauss points for the non-linearity.

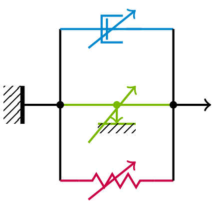
- For a volume element in large transformation, 9 components of stress gradient are observed. .adofi=zeros(18,1)+.99 will interlace 18 internal states with the observation at each Gauss point.

By default, one should let initialization procedures allocate DOF identifiers to each internal states by only providing a DOF extension in .adofi. If internal states have a physically defined nodal support, it is also possible to provide the corresponding DOF instead of just an extension. Beware that these DOF should not be coupled to the elastic model, as external resolution would interfere

with the internal dynamics. Internal DOF replication for each Gauss point is automatically carried out if a single column is provided.
To allow clean representation and access to internal states, the global model DOF are automatically augmented with DOF associated to internal states. One can then decide whether to keep them or not during the resolution phase by setting positive (kept) or negative (eliminated) signs to the non-empty values in `.adofi`. To simplify for a given non-linearity all internal states are either kept or not, any negative value will then switch to elimination. In the case where internal states are kept during the resolution displacement and velocity states are automatically updated in the model.
`HBM solvers specificity`
HBM formulations have a resolution approach that is different from usual transient simulations that usually require to write separate dedicated functions for both resolution strategies.
In transient simulations, strain history is available, so that one first integrates internal states defined by equation (1.20), and then computes the non-linear forces with equation (1.19).
In HBM based formulations the steady state response state is assumed in the prediction/correction scheme, including strain history. Internal states coefficients are thus predicted, so that the corresponding non-linear force defined inequation (1.19) is directly obtained. One then updates the internal states evolution equation (1.20) for convergence iterations. Internal states DOF must thus be kept in the resolution phase.

1.2.7 Hyper-visco-hysteretic 0D model

For the representation of bushings, each Gauss point is a scalar (0D) constitutive law that considers a combination of hyperelastic (red curve in figure 1.2), rate independent hysteresis (green curves from low speed triangular testing), and viscoelastic behavior (blue to yellow maps denoting frequency and amplitude dependence from sine testing).



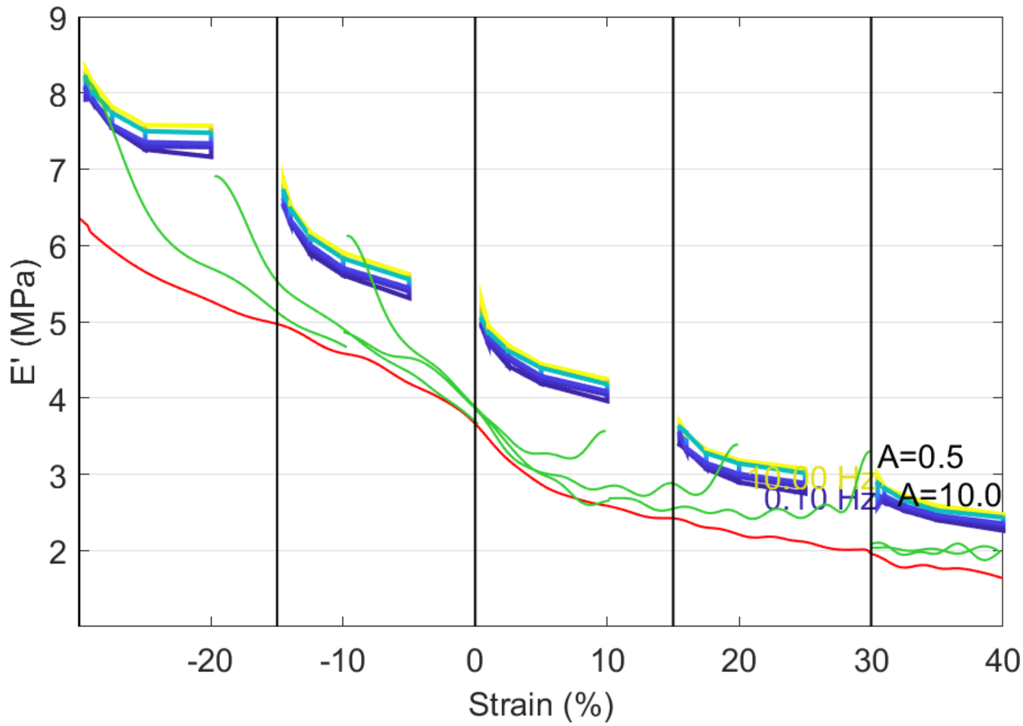


Figure 1.2: Tangent hyperelastic stiffness (red), hysteretic relaxation modulus (green), viscoelastic complex modulus (blue to yellow)

The total load is a series of forces/stresses

$$F = \sum_{i=0}^{N_{cell}} F^i \tag{1.22}$$

where hyperelasticity is represented using a tabular form with either linear of piecewise cubic interpolation with no internal states $F_g^0(u_g)$ (stored as `NLdata.Fu{k}`)and other behavior is represented using a series of cells (also called branches in reference to the classical rheologic representation of figure 1.3), specified using a field `.cell` where each row describes a branch indexed by i with `[ty g_i f_i xf_i a_i]` type, load fraction, and if used frequency in Hz , sliding distance, non-linearity coefficient. Currently supported types

- `1` viscoelastic relaxation of strain, using one internal state u_g^i associated with each Gauss point, a relaxation frequency $f_i = \omega^i/2\pi = K^i/(c^i 2\pi)$ and a load fraction $g_i = K^i/K^\infty$ (stored in a `NLdata.Fu{k}.cell` row containing `[1 gi fi(Hz) 0]`)

$$\frac{\dot{u}_g^i}{\omega_i} = (u_g - u_g^i) \text{ and } F^i = (K^\infty g^i)(u - u^i) = (K^\infty g^i)\left(1 - \frac{1}{s/\omega^i + 1}\right) u_g \tag{1.23}$$

Note that F_g^0 should here be the low frequency asymptote.

- 2 viscoelastic relaxation of strain rate, internal state u_g^i

$$\dot{u}_g^i + \omega_i u_g^i = \dot{u}_g \text{ and } F^i = (K^\infty g^i) u^i = (K^\infty g^i) \frac{s u_g}{s + \omega^i} \tag{1.24}$$

- 3 viscoelastic relaxation of hyperelastic stress, internal state F_g^i

$$\frac{\dot{F}_g^i}{\omega_i} + F^i = -\frac{g^i}{g^0} F^0(u_g) \tag{1.25}$$

- 4 viscoelastic relaxation of hyperelastic stress rate, internal state F_g^i,

$$\dot{F}_g^i + \omega_i F^i = \frac{g^i}{g^0} \dot{F}^0(u_g) = \frac{g^i}{g^0} \left.\frac{\partial F^0}{\partial u}\right|_{u_g} \dot{u}_g \tag{1.26}$$

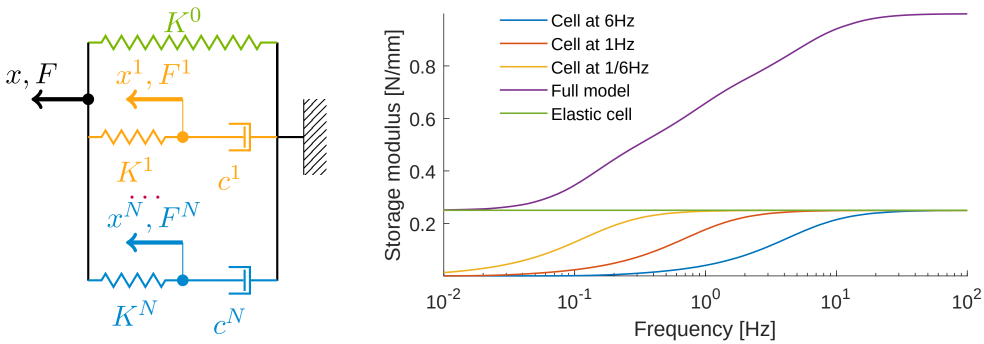


Figure 1.3: Generalized Maxwell model and associated frequency domain response XXX change legend 1/tau1=6Hz

- 8 Lion transition between viscous and hysteretic behavior xxx

$$\dot{F}_g^i + F_g^i \left(\omega_i + \frac{|\dot{u}_g|}{x_f^i}\right) = \frac{g^i}{g^0} \dot{F}^0 \tag{1.27}$$

- 9 Lion transition between viscous and hysteretic behavior *for 3D deviatoric stress* $\|\dot{u}\|$ is a scalar per material point and not by stress component.

$$\dot{F}_g^i + F_g^i \left(\omega_i (1 + \beta_i \|\dot{u}\|)\right) = \frac{g^i}{g^0} \dot{F}^0 \tag{1.28}$$

- 5 friction element (Iwan model, Jenkins cell) with simple target, parameters are sliding distance $x_f^i = F_f^i/K^i$ and load fraction $g_i = K^i/K^\infty$. The load saturation formulation with internal state corresponding to the hysteretic force F_g^i and evolution equation

$$\begin{array}{ll} \dot{F}_g^i = \dfrac{g^i}{g^0}\dot{F}^0, \text{ if } F_g^i \text{sign}(\dot{u}_g) < F_f^i = g_f^i F^0(u_g) = xxxwasK^i x_f^i & \text{sticking state} \\ \dot{F}_g^i = 0, \text{ if } F_g^i \text{sign}(\dot{u}_g) = F_f^i & \text{sliding state} \end{array} \tag{1.29}$$

is preferred to the classical displacement formulation with internal state u_g^i position of last turning point,

$$\begin{array}{ll} \dot{u}_g^i = 0, \text{ if } \left\|u_g - u_g^i\right\| < x_f^i = \frac{F_f^i}{K^i} & \text{sticking state} \\ \dot{u}_g^i = \dot{u}_g, \text{ if } \left\|u_g - u_g^i\right\| = x_f^i = \frac{F_f^i}{K^i} & \text{sliding state} \end{array} \tag{1.30}$$

- 6 friction element with Dahl transition using internal state F_g^i, parameters $F_f^i = K^\infty g_i x_f^i$ limit force and speed exponent α^i (stored in `a_i` coefficient) associated with evolution equation

$$\dot{F}_g^i = \left(1 - \frac{F_g^i}{(g_f^i/g^0)F^0}\text{sign}(\dot{u}_g)\right)^{\alpha_i} \frac{g^i}{g^0}\dot{F}^0, \tag{1.31}$$

- 7 friction element with Dahl stress relaxation using internal state F_g^i, parameters $F_f^i = K^\infty g_i x_T^i$ limit force and speed exponent α^i (stored in `NLdata.Fu{k}.g`, `.xf` and `.aDahl`) xxx associated with evolution equation

$$\dot{F}_g^i = (K^\infty g_i)\left(1 - \frac{F_g^i}{F_f^i}\text{sign}(\dot{u}_g)\right)^{\alpha_i} \dot{u}_g, \tag{1.32}$$

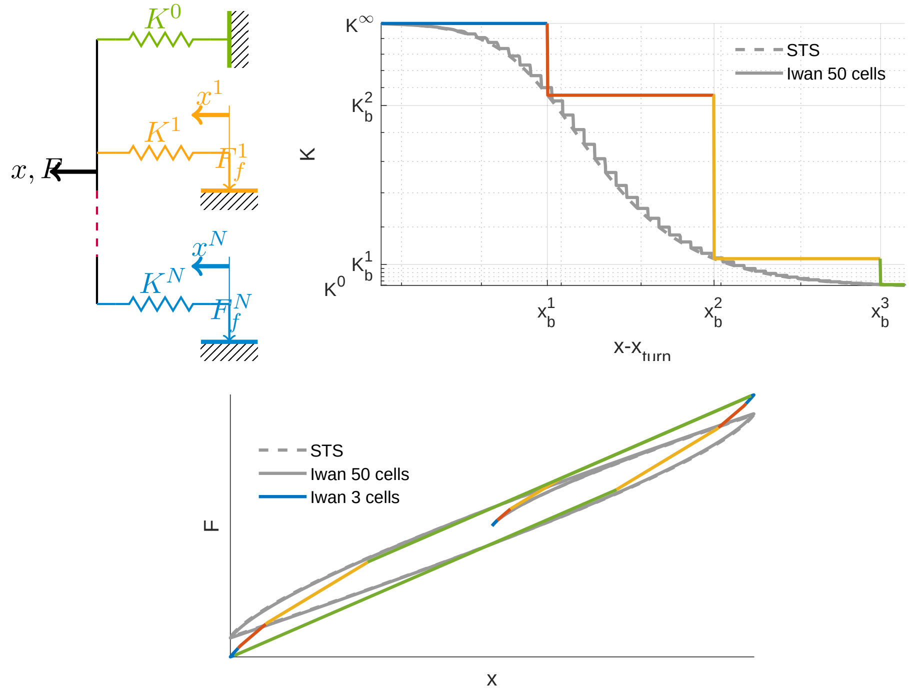


Figure 1.4: Scheme for Iwan model and respective response in terms of hysteretic relaxation and on force/displacement domain.

For runtime integration, the data necessary for evolution equations is stored as

- `.opt=[ FuCode=uMax tc dt0 ]` followed by a series of cell constants starting at opt(10).
- `.iopt` starts with the standard `Find, iu, Nstrain, Nistate, Ngauss(5)`, then a list of operations are specified starting from `tcell(10)` and `cval=opt(10)`.
  - `tcell=-1,cval=[]` last cell
  - `tcell=1,cval=[v1 v2 v3]` viscoelastic (Maxwell) relaxation of strain, 3 `cval` values give the recursion $u^i := (u^i v_2 + u^0)/v_1$ and load fraction definition $F^i = v_3 u^i = K^i u^i$. $v_1 = \omega_i$ in rad/s, $v_2 = g_i/g_0$, if non-zero $v_3 = 1/x_{fi}$.
  - `2, cval=[v1 v2 v3]` viscoelastic relaxation of strain, values in `cval` give the recursion $u^i := (u^i v_2 + v^0)/v_1$ and gain $F^i = v_3 u^i = K^i u^i$
  - `3, cval=[v1 v2 v3]` viscoelastic relaxation of stress, values in `cval` give the recursion $F^i := (F^i v_2 + v_3 F^0)/v_1$.

– `4, cval=[v1 v2 v3]` viscoelastic relaxation of stress rate, values in `cval` give the recursion $F^i := (F^i v_2 + v_3 \dot{F}^0)/v_1$. xxxLion saturation needs documenting
– `7` xxx need to document friction xxx
– `100 step` increment direction.
– `101 F0_dim1 F0_dim2` interpolation table to estimate F^0 for dim2==2 piecewise cubic interpolation is used.
– `102` linear scalar stiffness, one constant
– `103` linear scalar viscosity
– `tcell=[1000 unlshift snlshift]` component change.
– `tcell=300, cval=[c1, c2, c3, kappa, kappav]` hyperelastic material cell `he MooneyRivl CiarletGeymonat`
– `tcell=301, cval=[omegai,gi/g0,ef]` relaxation of deviatoric part of S given by $\dot{S}^i + (\omega^i + \frac{\|d\|}{\epsilon_f})S^i = \frac{g^i}{g^0}\dot{S}^0$ xxx discuss (2.39) with Rafael xxx
– `tcell=[400 ]` xxx non usual interlacing Ng for first strain g, Ng of u_{t1}, u_{t2},

The constants v_i depend on the integration scheme and constitutive laws

- For implicit integration of Maxwell form (1.23), the evolution equation of the internal state is

$$u^i(t_{n+1})\left(\frac{1}{\omega^i dt}+1\right) = \frac{u^i(t_n)}{\omega^i dt} + u(t_{n+1}) = u^i(t_{n+1})v_1 = u^i(t_n)v_2 + u(t_{n+1}) \tag{1.33}$$

- For implicit integration of Maxwell form (1.24), the evolution equation of the internal state is

$$u^i(t_{n+1})\left(\frac{1}{dt}+\omega^i\right) = \frac{u^i(t_n)}{\delta t} + \dot{u}(t_{n+1}) = u^i(t_{n+1})v_1 = u^i(t_n)v_2 + \dot{u}(t_{n+1}) \tag{1.34}$$

- For implicit integration of Maxwell form (1.25), the evolution equation of the internal state is

$$F^i(t_{n+1})\left(\frac{1}{\delta t}+\omega^i\right) = \frac{F^i(t_n)}{dt} xxx\dot{u}(t_{n+1}) = u^i(t_{n+1})v_1 = u^i(t_n)v_2 + \dot{u}(t_{n+1}) \tag{1.35}$$

  xxx

1.2.8 Hyper-visco-hysteretic 3D model

The implementation of bushing also supports 3D constitutive laws xxx

1.2.9 Maxwell cell model using matrices (deprecated)

The Maxwell cell model belongs to a category of so-called *rheology* based models. the force at each Gauss point is calculated based on an internal rheology identified as a spring-mass based model. Figure 1.5 illustrates the Maxwell (or Zener) model.

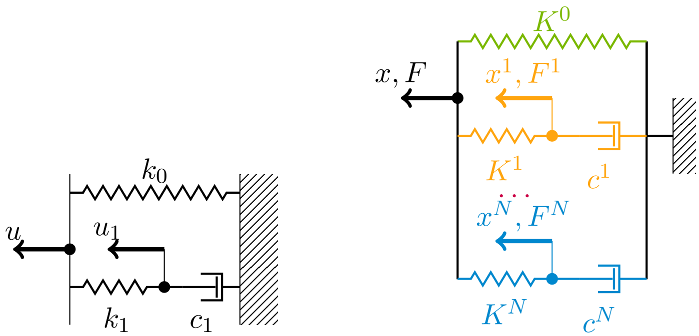


Figure 1.5: Standard viscoelastic model (single cell Maxwell or Zener). Generalized multi-cell model.

Since each branch is decoupled, one can write a series of scalar internal state evolution equations

$$\dot{e}_n = \frac{K_n}{C_n}(e_n - e_b) \tag{1.36}$$

and recompose the total non-linear stress using

$$s_{nlb} = K_0 e_b + \sum_n K_n (e_b - e_n) \tag{1.37}$$

To allow more general superelement representation of the constitutive law, the external force s_{nlb} can be defined defined by a first order equation involving the system strain state at a given Gauss point e_b and $\dot{e}_b$, and a series of internal states $\mathbf{e}_i$ and $\dot{\mathbf{e}}_i$. In the following bold letters refer to non-scalar values for a given Gauss point.

$$\begin{bmatrix} K_{bb} & \mathbf{K}_{bi} \\ \mathbf{K}_{ib} & \mathbf{K}_{ii} \end{bmatrix} \begin{Bmatrix} e_b \\ \mathbf{e}_i \end{Bmatrix} + \begin{bmatrix} C_{bb} & \mathbf{C}_{bi} \\ \mathbf{C}_{ib} & \mathbf{C_{ii}} \end{bmatrix} \begin{Bmatrix} \dot{e}_b \\ \dot{\mathbf{e}}_i \end{Bmatrix} = \begin{Bmatrix} s_{nlb} \\ \mathbf{0} \end{Bmatrix} \tag{1.38}$$

No external forces being applied on the internal rheology, one is able to use a Schur complement to obtain the internal states evolution equation

$$\dot{\mathbf{e}}_i = -\mathbf{C}_{ii}^{-1}\mathbf{K}_{ib} e_b - \mathbf{C}_{ii}^{-1}\mathbf{C}_{ib}\dot{e}_b - \mathbf{C}_{ii}^{-1}\mathbf{K}_{ii}\mathbf{e}_i \tag{1.39}$$

Once the internal state is resolved, the external force can be deduced

$$s_{nlb} = \begin{bmatrix} K_{bb} & \mathbf{K}_{bi} \end{bmatrix} \begin{Bmatrix} e_b \\ \mathbf{e}_i \end{Bmatrix} + \begin{bmatrix} C_{bb} & \mathbf{C}_{bi} \end{bmatrix} \begin{Bmatrix} \dot{e}_b \\ \dot{\mathbf{e}}_i \end{Bmatrix} \tag{1.40}$$

Resolution of equation (1.39) can be obtained with different strategies. In transient simulations a local integration is performed using the Euler scheme. From instant t_n the internal state at instant t_{n+1} has to be integrated over $h = t_{n+1} - t_n$

$$\mathbf{e}_i^{n+1} = \mathbf{e}_i^n + h(1-\theta)\dot{\mathbf{e}}_i^n + h\theta\dot{\mathbf{e}}_i^{n+1} \tag{1.41}$$

Implemented strategies can involve either single or multiple step integration in implicit or explicit form. Implicit resolution involves Newton-Raphson resolution, the usual initial prediction assuming constant velocity over the time step.
In the harmonic balance method, the global resolution scheme iterates on the internal states, so that one directly computes equations (1.39) and (1.40) from the current solution and the be used in the residue equation. The harmonic balance scheme then checks non-linear forces equilibrium and the stability of internal velocities.

1.3 Jacobian computations

See `nl_spring` `NLJacobianUpdate`.

1.4 Transient solution of non-linear equations

1.4.1 Principles

To allow the use SDT transients with external FEM packages, it is assumed that a superelement representation of the model is imported

$$[M_R]\{\ddot{q}_R(t)\} + [C_R]\{\dot{q}_R(t)\} + [K_R]\{q_R(s)\} = [b_R]\{u(t)\} \tag{1.42}$$

In general, the reduction is performed so that the DOFs retained $\{q_R\}$ are related to the original DOFs of a larger model by a Rayleigh Ritz reduction basis T using

$$\{q\}_N = [T]_{N\times NR}\{q_R(s)\}_{NR} \tag{1.43}$$

This representation is fairly standard. The data structure representation within SDT is described in section 1.7.1 . SDT/FEMLink supports import from various FEM codes and more details are given in section 2.4.2 for NASTRAN, section **??** for Abaqus, and section **??** for ANSYS.
For transient resolution a real representation of damping must be used. Rayleigh and viscous damping are thus the only solutions supported. It is noted that for sine sweeps, it is possible to consider a time varying Rayleigh damping which has been found to be appropriate in some cases.

nl_solve fe_timeModalNewmark implements an optimized fixed time step version of the Newmark scheme (see [?] section 4.1.4) assuming a modal basis associated with the underlying linear system (discussed in section 1.4.3).
The non-linear resolution of the mechanical equation is usually performed by an iterative predictor/corrector scheme. Given the solution at time step n, the prediction is initialized by assuming a null acceleration at time step $n+1$, so that the predictors q^0_{n+1} and $\dot{q}^0_{n+1}$ are expressed as

$$\left\{ \begin{array}{l} q^0_{n+1} = q_n + h\dot{q}_n + h^2(\frac{1}{2} - \beta)\ddot{q}_n \\ \\ \dot{q}^0_{n+1} = \dot{q}_n + h(1-\gamma)\ddot{q}_n \end{array} \right. \tag{1.44}$$

One considers the displacement correction Δq_{n+1} as the only unknown and velocity and acceleration at time step $n+1$ are given by

$$\left\{ \begin{array}{l} \Delta q_{n+1} = q_{n+1} - q^0_{n+1} \\ \\ \dot{q}_{n+1} = \dot{q}^0_{n+1} + \frac{\gamma}{h\beta}\Delta q_{n+1} \\ \\ \ddot{q}_{n+1} = \frac{1}{\beta h^2}\Delta q_{n+1} \end{array} \right. \tag{1.45}$$

Provided solution q^k_{n+1}, the residue is defined as

$$r^{k+1}_{n+1} = [M]\, \ddot{q}^k_{n+1} + [C]\, \dot{q}^k_{n+1} + [K]\, q^k_{n+1} - f_{cn+1} - f_{NLn+1}(q^k_{n+1}, \dot{q}^k_{n+1}, t_{n+1}) \tag{1.46}$$

and the correction is found by solving $J\Delta q^{k+1}_{n+1} = r^{k+1}_{n+1}$ using the diagonal fixed Jacobian

$$J = \left[\diagdown \frac{1}{\beta h^2} + \frac{\gamma 2\zeta_j\omega_j}{\beta h} + \omega_j^2 \diagdown \right] \tag{1.47}$$

For one step formulation see [?] formula (4.53).
For a given system, a one-step Newmark is the combination of a linear evolution matrix depending on the linear system properties and time step h , and external forces. One thus writes the discrete state evolution equation as

$$\left\{ \begin{array}{c} q_{n+1} \\ \dot{q}_{n+1} \end{array} \right\} = [E(h)] \left\{ \begin{array}{c} q_n \\ \dot{q}_n \end{array} \right\} + \{f_{ch}\} \tag{1.48}$$

The evolution equation combines the quadrature rules and the mechanical equilibrium at states n and $n+1$:

$$\left\{ \begin{array}{l} q_{n+1} = q_n + h\dot{q}_n + h^2\beta\ddot{q}_{n+1} + h^2\left(\frac{1}{2} - \beta\right)\ddot{q}_n \\ \dot{q}_{n+1} = \dot{q}_n + h\gamma\ddot{q}_{n+1} + h\left(1-\gamma\right)\ddot{q}_n \\ M\ddot{q}_{n+1} + C\dot{q}_{n+1} + Kq_{n+1} = f_{cn+1} \\ M\ddot{q}_n + C\dot{q}_n + Kq_n = f_{cn} \end{array} \right. \tag{1.49}$$

Multiplying the quadrature equations by M and replacing acceleration terms by their mechanical equation resolution provides the evolution equation that can be written in matrix form

$$\begin{bmatrix} M+h^2\beta K & h^2\beta C \\ h\gamma K & M+h\gamma C \end{bmatrix} \begin{Bmatrix} q_{n+1} \\ \dot{q}_{n+1} \end{Bmatrix} = \begin{bmatrix} M-h^2\left(\frac{1}{2}-\beta\right)K & hM-h^2\left(\frac{1}{2}-\beta\right)C \\ -h\left(1-\gamma\right)K & M-h\left(1-\gamma\right)C \end{bmatrix} \begin{Bmatrix} q_n \\ \dot{q}_n \end{Bmatrix} + \cdots \\ \begin{Bmatrix} h^2\beta f_{cn+1} + h^2\left(\frac{1}{2}-\beta\right)f_{cn} \\ h\gamma f_{cn+1} + h\left(1-\gamma\right)f_{cn} \end{Bmatrix} \tag{1.50}$$

The evolution matrix is then

$$[E(h)] = \begin{bmatrix} M+h^2\beta K & h^2\beta C \\ h\gamma K & M+h\gamma C \end{bmatrix}^{-1} \begin{bmatrix} M-h^2\left(\frac{1}{2}-\beta\right)K & hM-h^2\left(\frac{1}{2}-\beta\right)C \\ -h\left(1-\gamma\right)K & M-h\left(1-\gamma\right)C \end{bmatrix} \tag{1.51}$$

and the interpolated external force is then

$$\{f_{ch}\} = \begin{bmatrix} M+h^2\beta K & h^2\beta C \\ h\gamma K & M+h\gamma C \end{bmatrix}^{-1} \begin{Bmatrix} h^2\beta f_{cn+1} + h^2\left(\frac{1}{2}-\beta\right)f_{cn} \\ h\gamma f_{cn+1} + h\left(1-\gamma\right)f_{cn} \end{Bmatrix} \tag{1.52}$$

The acceleration can then be resolved with one of the quadrature rules, the simplest being the velocity quadrature providing the relation

$$\ddot{q}_{n+1} = \frac{1}{h\gamma}\left(\dot{q}_{n+1} - \dot{q}_n\right) - \frac{1-\gamma}{\gamma}\ddot{q}_n \tag{1.53}$$

1.4.2 Enforced displacement, resultants

When considering enforced displacement, the residual at a given time is

$$\begin{Bmatrix} R_I \\ R_C \end{Bmatrix} = \begin{bmatrix} M_{II} & M_{IC} \\ M_{CI} & M_{CC} \end{bmatrix} \begin{Bmatrix} \ddot{q}_I(t) \\ \ddot{q}_C(t) \end{Bmatrix} + \left\{ F_{NL}\left(\begin{Bmatrix} q_I \\ q_C \end{Bmatrix}, \begin{Bmatrix} \dot{q}_I \\ \dot{q}_C \end{Bmatrix} \right) \right\} - [b_{Ext}]\{u(t)\} \tag{1.54}$$

Different strategies are implemented to compute the enforced time derivatives of q_I. Simple approaches uses the base definition of curves for each column of `bset.def` which corresponds to

$$\begin{Bmatrix} \ddot{q}_I & \dot{q}_I & q_I \end{Bmatrix} = \{T_I\}\begin{Bmatrix} \ddot{u}_I(t) & \dot{u}_I & u_I \end{Bmatrix} \tag{1.55}$$

Observation of the residual loads on the enforced part requires the R_I part of the residual to be computed, this prevents the use of an elimination strategy before residual computations and the elimination must thus be performed, when solving for q_c and its time derivatives. Resultant on a fixed body associated with the sum of an arbitrary set of non-linearities thus requires an enforced zero displacement (`DofSet` and not `FixDof` entry).

Non-linearities may also lead to resultant like observations (the transmitted load), but this is then considered to be a generalized form of stress so that the quantity of interest must be exported as a stress or an internal state during time integration.

1.4.3 Definition of an underlying linear system

When defining a non-linear constitutive law, it is always possible and often desirable to define an underlying linear system. Taking the simple case of a cubic spring where $s_{nl} = e_{nl}^3$. Figure 1.6 clearly illustrates the difference between the tangent stiffness, slope of force at current point $3e_{nl}^2$, and the secant stiffness, ratio of force divided by deformation e_{nl}^2.

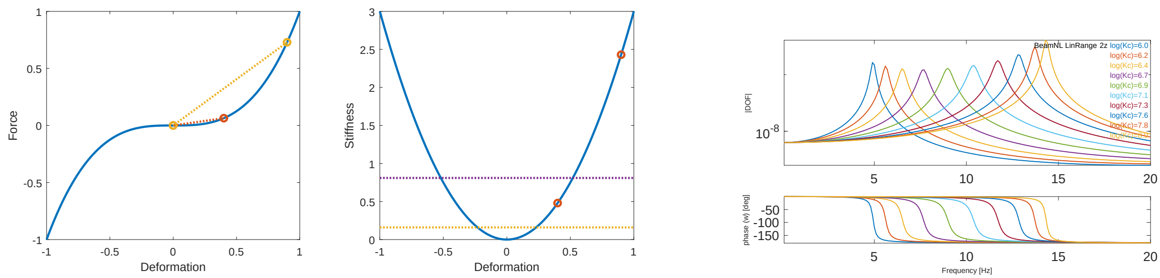


Figure 1.6: Left : Tangent and secant stiffness. Right : possible underlying linear systems for the `BeamNL` example.

When defining a non-linear constitutive law, it useful and SDT-HBM requires that an underlying linear system be defined. For a general $F_{nl}(e(t), \dot{e}(t))$ law, the non-linear stress used in time integration should thus be of the form

$$\{s_{nl}(t)\} = \{F_{nl}(e(t), \dot{e}(t))\} - [k_J]\{e_{nl}(t)\} - \{F_0\} \tag{1.56}$$

with $[k_J]\{e_{nl}(t)\}$ the chosen linear representation of the non-linearity and F_0 the value of the non-linear stress at the system state around which the response is computed.
When considering assembly in SDT, elements with a non-linearity defined through the `NLdata` field are ignored in linear assembly if `NLdata.keepLin=0`. For example, for a Maxwell model, reduction is best performed using the high frequency modulus. Thus a non-linear spring should be coupled with a linear spring using that high stiffness.

1.5 Data structures for non-linearities in time

1.5.1 `NLdata` non-linearity definition (model declaration)

A non linearity has the following input form when stored as `NLdata` of an element group property.

- `.type` a MATLAB function handle which is used for initialization phases. The most commonly used non-linearity is `nl_inout`. For transient simulations observation matrices are built in `nl_spring('init')` while for HBM solutions this is done in `hbm_solve InitHBM`.

- `.Fu` can be done through generic anonymous functions see section 1.2.5 , user defined anonymous functions see section 1.2.4 , tabular definitions section 1.2.2 .

- `.Fv` can be used for tabular definitions section 1.2.2 of laws on velocity.

- `.adof` optional definition of DOF specific to the non linearity that will be propagated to `model.FNLDOF`. For vectorized non-linearities, this gives the decimal part describing fields associated with `.snl`. To associate other DOF to `.unl`, you can set a `.udof` field.

- `.isens` optional : may be used to select a partial list of strains normally computed. For example to only keep translations of a `cbush` use `.isens=[1 2 3]`.

- `.adofi` to declare internal states, in coherence with field `.MatTyp` below. This field can be omitted of left empty if no such feature is used. For the cell array format, rows correspond to each observed strain, and multiple columns can be given when multiple Gauss points differ. More details are given under `DOF representation of internal states`.

- `.keepLin` set to zero to avoid assembly of linear behavior in the base matrices

- `.MatTyp` : declares the time derivative of the signal associated to each internal DOF. When using explicit definitions of internal DOF, as in HBM or time integration with explicit internal states, this is used to specify how to compute the Jacobian (see `nl_spring NLJacobianUpdate`). Thus `MatType=1` stiffness corresponds to a displacement DOF, `2` mass an acceleration DOF, `3` viscous damping a velocity DOF.

- `.Jacobian` can be a function computing the tangent matrix or a number as described for `nl_spring nl_inout`.

- `.obs` Optional command to fill in `out.FNL.NL`, see below. This can either be a logical, evaluating `nl_fun('obs')`, or a cell-array giving non linear data fields to propagate to the output from `model.NL`, or a string which will be evaluated.

- `.snl` as a empty field will force the initialization of a `NL.snl` buffer during time integration.

- `.StoreType` can be used to specify the `StoreType` strategy.

- `.unl(:,:,2)=unl0` Optional. Define an offset in observation (before applying Fu) as $unl = c * u + unl0$. It can be a vector giving direct offset (as many lines as c). It can also be a string defining what offset to apply : the only strategy implemented at this time is `unl0='q0'` to remove observation of the static from observation at each time step. It can also be the name of a curve stored in model stack.

1.5.2 Additional fields in model

Base on model stack entries of the `pro` type where as `NLdata` field is present as described in the previous section, `nl_spring` support additional model fields.

- `.NL` This is a stack containing all non linearity data built using `nl_spring NL`. The stack has as many lines as existing non-linearities and three columns, the first being the non linearity type (the non linear function name most of the time), the second the non linearity name, and the non linearity data described in section 1.5.3 .
- `.FNL` This is a transfer vector containing non linear data to output. Each non-linearity can store output data based on `.Find`.
- `.FNLDOF` This is a DOF vector corresponding to vector FNL. These FNLDOF are defined freely and not critical for base functionality. They are however convenient to keep track of internal states or non-linearity output, so that its building should be taken care of.
- `.FNLlab` This is a cell array of labels corresponding to non linear data to output. This is used by `iiplot` when non-linear responses are displayed during cleanup operations.

Output from simulations with non-linearities, from `nl_solve`, or `fe_time`have additional data stored in a `.FNL` field. This is a deformation structure with fields

- `.def` Stored content of `model.FNL` at saving times.
- `.DOF` Stored content of `model.FNLDOF`.
- `.data` Copied data of out.data (saved times)
- `.lab` Stored content of `model.FNLlab`.
- `.NL` This is a cell array of the same format than `model.NL`. The third column contains structures containing non linear data relevant for result display. By default it contains field `.iNL`, the index vector of the current non linearity in the full `FNLDOF`. This field can be modified using the `.obs` callback of the non linearities. This field is based on `NL.FInd`, if `.FInd` is missing, `iNL` will be empty. `iNL` is then generated using the length of `NL.adof`, or by default using the length of `NL.unl`.

1.5.3 NL structure : non-linearity representation during time integration

During time integration, non-linearities are stored in the `model.NL` cell array. Each non-linearity is a data structure with the following standard fields for optimized computation (see `mkl_utils`

- `.type` a MATLAB function handle which is then called through `NL.type(NL,fc,model,u,v,q,opt,Case)`. Some older NL use a string giving the type name.
- `.c` observation matrix (1.3) for non linear displacements and velocities. During solves this is stored in row format obtained with `NL.c=v_handle('mklst',sparse(NL.c))` to optimize product speed.
- `.unl` pre-allocated memory for the result of `NL.c*u`. Must be consistent with the number of rows in `NL.c`. The computation is handled by `mkl_utils`. New implementations support a third dimension to store `.unl0` in `.unl(:,:,2)` and `.unlj1`, the value at the previous time step, in `.unl(:,:,3)`.
- `.vnl` if exists pre-allocated memory for the result of `NL.c*v`. Must be consistent with the number of rows in `NL.c`. The computation is handled by `mkl_utils`.
- `.snl` preallocated buffer of length `size(b,2)` to store the non-linear stresses (1.6).
- `.b` command matrix for non linear loads. At the end of the `NL.type` call it is expected that `NL.snl` (which may point to `NL.unl` if buffer overwrite is acceptable) contains the non linear component loads such that the residual becomes `r=r+NL.b*NL.snl`.

  During solves, sparse matrix operations must be optimized. This the product bs_{nl} is computed, it is interesting to store the transpose of the b matrix. The matrix is expected to be in transposed form `NL.b=v_handle('mklst',b);`. This conventions allow reuse of a `.c` matrix for command.

  Note that the sign conventions when using `unl` to return a non linear force are opposite to what is done when the result is added to `fc`, see `sdtweb nl_fun` to compare conventions.
- `.FInd` C++ start index in `model.FNL` to store the current non linearity data.
- `.adof` FNLDOF specific to the non linearity.
- `.obs` Optional command to fill in `out.FNL.NL`, see below. This can either be a logical, evaluating `nl_fun('obs')`, or a cell-array giving non linear data fields to propagate to the output from `model.NL`, or a string which will be evaluated.
- `.opt` data vector containing *double* values to be used in C++ implementation of non-linearities. Default values would be `FuCode(1) tc(2) dt0(3) val1 ...`

  - `val1, ...` is a `FuCode` dependent storage
  - for tables the standard, see `sdtweb('d_fetime','stdFuToOpt')` is to use `iopt[6:3:.]` to point to start of table in `.opt` stored, give `M` as `[(double)jcur,Val/Type,s, table]`

- `.iopt` int32 data vector containing non-linearity specific integers. `(1)FInd` C start of model.FNL, `(2)iu` C staring position of internal states in global displacement vector, `(3)Nstrain` number of observed strains for each Gauss point. `(4)Nistate` number of internal states per Gauss point. `(5)Ngauss` number of Gauss points in a vectorized non-linearity. Later format may be dependent on non-linearity, a few variants are documented below to improve standardization

  - `FuTable` no internal state `iopt(6)=xStartC,(7)yStartC, (8)size(y,1), (9)size(y,2)` C style start in `.opt` of table with `unl` as first column and values in second followed by `cur` (3 values). When combined with `FvTable iopt(10)=xStartC,(11)yStartC, (12)size(FvY,1), (13)size(FvY,2)`.
  - constitutive laws with internal states will use `iopt(6)StoreType` update strategy in residual call, `iopt(7:9)` formulation dependent choices. `iopt(10)=tstartC (11)size(y,1), (12)size(y,2)` may be used for tabular definitions (hyperelasticity for example) where table starts by 3 values `cur,val,s` followed by the x vector and y matrix.

- when calling MATLAB from the mex, `.MexCb` is a cell array giving callback function and options. For optimized operation without fieldname checks, the option structure is assumed to have ordered fields : `.unlg` at a single Gauss point, `.opt` copy of `NL.opt`, `.jg` int32 vector allowing passing of current Gauss point index, `.vnlg` optional copy of velocities at gauss point or empty. See section 1.2.3 .

- in the C implementations the N field gives `[0]numel(unl(:,:,1)) [1]size(c,1) size(c,2) isTrans(c) [4]size(b,1) size(b,2) [6]isTrans(b) [8]length(opt) length(iopt) [10]siz`

- **obsolete**

- `.extDOF` obsolete see iopt, double vector containing Matlab indices `[iu,Nstrain,Nistate]`

- Obsolete `.unl0` offset to apply to `.unl`, so that the content of `.unl` becomes `NL.c*u+unl0`. *Current implementations* should store in `.unl(:,:,2)`.

- Obsolete `.vnl0` offset to apply to `.vnl`, so that the content of `.vnl` becomes `NL.c*v+vnl0`. Current implementations should store in `.vnl(:,:,2)`.

1.6 Harmonic balance solutions and solver

1.6.1 Time/frequency representation of solutions/loads

The solver is meant to support a generic representation of time dependence. The generalized degrees of freedom considered here are components of a vector noted Z and correspond to amplitudes multiplying functions of both space and time. For a scalar function of space (single spatial DOF), multiple harmonic DOF f_k and time varying shape functions $h_k(t)$ fully describe the time dependence

$$\{f(t)\} = \sum_{k\in\mathcal{H}} f_k h_k(t) = \{f_k\}_{n_k}^T \left[H_{kt}\right]_{n_k\times n_t} \tag{1.57}$$

In the specific case of harmonic balance [?], the time dependence of a given DOF is written as

$$q(t) = z_{cq0} + \sum_{n\in\mathcal{H}} \left(z_{sqn}\ sin\left(n\omega t\right) + B_{cqn}\ cos\left(n\omega t\right)\right) = \sum_{k\in\mathcal{H}} Z_{qk} h_k(t) \tag{1.58}$$

where spatial DOFs are indexed by q and temporal DOFs are indexed by k but need also a representation of the harmonic n. Thus the ordering of generalized space/time DOFs is

$$\{Z_{qk}\} = \left\{\begin{matrix} z_{cq0} \\ z_{sq1} \\ z_{cq1} \\ \vdots \end{matrix}\right\}_{n_q.n_h\times 1} \tag{1.59}$$

with a spatial dependence q (degree of freedom in the time domain equation(1.1)) and a temporal dependence k. Note that in the notation above, the individual components are assumed real, but the problem can also be written in complex form (1.71). As always in SDT, it is expected that the ordering of Z could be changed. Thus a list of DOFs is needed to specify the meaning of each component of the vector. This list is given in the form

```
hdof={ % List of active DOFs
  1.01   'c0'  % Node1.DOF1(x), constant(B0)
 10.02   's1'  % Node10.DOF2(y), first harmonic (A1)
 10.02   'c1'  % Node10.DOF2(y), first harmonic (B1)
};
```

where the first column specifies the spatial DOF and the label in the second column the temporal component. `hbm_solve harm` commands provide utilities to manipulate spatio-temporal DOF definitions.
The definition of a solution is thus made using a data structure with fields

- `.def` matrix with columns being Z_{qk} values in given configuration (frequency dependence will be obtained with multiple columns).

- `.hdof` cell array giving the meaning of spatio-temporal amplitudes in `.def` rows (see equation (1.58))
- `TR` since, in general harmonic balance is performed using a reduced model, restitution of the full DOFs is based on (2.6) with the reduction basis stored in `def.TR`.

Building of the time response of a given DOF $q(t)$ should, using the convention of summed indices, actually be written as

$$q(t_j) = Z_{ql} H_{lj} \tag{1.60}$$

with $H_{lj} = H_l(t_j)$ the l^{th} harmonic function at time t_j. Times are assumed to be sampled into the regular time interval $[t_1 : t_N]$, so that H can be written in this case as a matrix

$$H_{lj} = \begin{bmatrix} \mathbb{1}(t_1) & \dots & \mathbb{1}(t_N) \\ \sin(\omega t_1) & \dots & \sin(\omega t_N) \\ \cos(\omega t_1) & \dots & \cos(\omega t_N) \\ \vdots & \dots & \vdots \\ \sin(k\omega t_1) & \dots & \sin(k\omega t_N) \\ \cos(k\omega t_1) & \dots & \cos(k\omega t_N) \\ \vdots & \dots & \vdots \end{bmatrix} \tag{1.61}$$

In further equations and in the code, the notation `Hkt`(H_{kt}) is used even though in the real form (with `opt.Opt.complex=0`) there are two l lines for each harmonic k (except for harmonic 0). Note also that when using harmonic fractions (for example in engines it is usual to use $N/2$ harmonics to represent a period over two revolutions), you should declare `opt.Opt.nu=2` and the `.harm` field will contain $k = 1/2, 1, ...$

An advantage of the retained definition is that it is not necessary to define all harmonics. Furthermore problems with sub-harmonics of the excitation frequency can also be considered and only require development of appropriate label handling methods to define sub-harmonic functions $h_k(t)$.

The inverse transform is the way to obtain the harmonic amplitudes from a time vector and is associated with the `Htk` linear operator

$$Z_{ql} = \{q(t_j)\}_{Nq \times Nt} [H_{tk}]_{Nt \times Nt} = \{q(t_j)\} \frac{2}{N} \begin{bmatrix} \frac{1}{2} & \sin(\omega t_1) & \cos(\omega t_1) & \dots \\ \vdots & \vdots & \vdots & \dots \\ \frac{1}{2} & \sin(\omega t_N) & \cos(\omega t_N) & \dots \end{bmatrix} \tag{1.62}$$

It is thus verified that $H_{kt} H_{tk} = [I]_{Nk \times Nk}$.

Some of the literature considers linear DOFs (including modal DOFs and some physical DOFs), non-linear DOFs (internal or physical). SDT-HBM does not ask the user for the nature of DOFs since non-linear DOFs can actually be deduced from the consideration of DOFs present in non-linear observation matrices and the notion was not found to be needed.

The spatial DOF nature (physical, modal, internal) is also not necessary. It is however useful to have automated procedures to assign an identifier for every spatial DOF. It is thus expected that generalized (modal) DOFs be assigned a node number at the time of superelement creation. To avoid viewing mistakes DOF associated with modes or internal states are affected to DOF 99 (of the form `NodeId.99`).
Finally, some non-linearities use internal states. As the HBM solver needs explicit access to these states, the `hbm_solve InitHBM` command performs a pre-processing step that affects a node number for each internal state so that they are present in q.

1.6.2 Harmonic balance equation (real DOF)

The base iterations are associated with a non-linear least squares minimization problem of the form

$$\min_Z \|\{R(Z,\omega,u)\}\| = \min_Z \|[A(\omega)]\{Z\} - [b]\{u_{ext}\} - [b_{nl}]\{s_{nl}(Z,\omega)\}\| \tag{1.63}$$

The contents of matrices $[A(\omega)]$, $[b]$, and $[b_{nl}]$ depend on the choice of time functions in (1.60).
With the states described in (1.58) using (1.61), the rows of the residue matrix correspond to the work equilibrium equation (1.1) integrated over a period. Thus the harmonic 0 (constant term) is given by

$$R_0 = \sum_{i=1}^{N}(M\ddot{q} + C\dot{q} + Kq - F_{nl}(q,\dot{q},q_{nl},\omega,t_i) - F_{ext}(\omega,t_i)) \tag{1.64}$$

The harmonic k leads to a sine (respectively cosine) contribution corresponding to an integral over the N time points of a period

$$R_{sk} = \sum_{i=1}^{N}\sin(k\omega t_i)(M\ddot{q}(t_i) + C\dot{q}(t_i) + Kq(t_i) - F_{nl}(q,\dot{q},q_{nl},\omega,t_i) - F_{ext}(\omega,t_i)) \tag{1.65}$$

Assuming states ordered with sine contributions first followed by cosine, the generalized Jacobian matrix $\frac{\partial R}{\partial Z}$ is composed of a linear part

$$A_{\mathcal{H}} = \begin{bmatrix} K & & & & & & \\ & K-(\omega)^2 M & -\omega D & & & & \\ & \omega D & K-(\omega)^2 M & & & & \\ & & & \ddots & & & \\ & & & & K-(k\omega)^2 M & -k\omega D & \\ & & & & k\omega D & K-(k\omega)^2 M & \\ & & & & & & \ddots \end{bmatrix} \tag{1.66}$$

and a series of non-linear contributions

$$J_{\mathcal{H}} = [b_{\mathcal{H}}]\frac{\partial\{s_{\mathcal{H}}(Z,\omega)\}}{\partial Z} = [b_{\mathcal{H}}]\frac{\partial\{s_{\mathcal{H}}(\epsilon_{\mathcal{H}},\omega)\}}{\partial \epsilon_{\mathcal{H}}}\frac{\partial\{\epsilon\}}{\partial Z} = [b_{\mathcal{H}}]\frac{\partial\{s_{\mathcal{H}}\}}{\partial \epsilon_{\mathcal{H}}}[c_{\mathcal{H}}] \tag{1.67}$$

The linear part of $A_{\mathcal{H}}$ in (1.66) is skew-symmetric. The negative sign is applied to the lines corresponding to the sine contributions. Using the notation (1.58), the negative sign is then located on the upper triangular part.
In the implementations derived from [?], the non-linear contributions are computed by finite differences (or possibly analytically although this is not yet implemented). And the inner loop iterations of the non-linear solver seeking the solution of (1.63) are of the form $\Delta Z = [J]^{-1} R$.
In direct frequency solvers by SDTools, one seeks to obtain the ideal single step convergence, where a sequant inter-harmonic stiffness $[K_{\epsilon\mathcal{H}}]$ is found such that

$$\{s_{\mathcal{H}}\} = [K_{\epsilon\mathcal{H}}]\{\epsilon_{\mathcal{H}}\} \tag{1.68}$$

The two have the notable major difference that in general $[K_{\epsilon\mathcal{H}}] \neq \frac{\partial\sigma}{\partial\epsilon}$. In other words secant and tangent stiffness are different matrices.
In the case of a scalar strain ϵ and a linear complex stiffness of the form $k(1+i\eta)$. The inter-harmonic coupling is block diagonal and of the form

$$K_{\epsilon\mathcal{H}} = \begin{bmatrix} k & & & & & & \\ & k & -k\eta & & & & \\ & k\eta & k & & & & \\ & & & \ddots & & & \\ & & & & k & -k\eta & \\ & & & & k\eta & k & \\ & & & & & & \ddots \end{bmatrix} \tag{1.69}$$

since $-i\sigma_s + \sigma_c = k(1+i\eta)(i\epsilon_s + \epsilon_c) = ik(\epsilon_s + \eta\epsilon_c) + k(\epsilon_c - \eta\epsilon_s)$.
Another form considers an extended Z with frequency stored as an additional component (this is achieved using `opt.Opt.fvar=1`)

$$\min_{Z_e} \|[A_e(\omega)]\{Z_e\} - b_e\| \tag{1.70}$$

In this case a last column $\partial R/\partial\omega$ is added to the Jacobian. For the last row, the ideal would be to compute $\frac{\partial\omega}{\partial Z}$ but the evaluation of this quantity is quite difficult, so that arc length techniques are used for continuation.

1.6.3 Complex formulation and equivalent stiffness

To clarify the notion of equivalent stiffness found at the harmonic balance solution, the real harmonic balance states (1.58) can be written in complex form expressing the time response as

$$q(t) = \Re\left(\sum_{k\in\mathcal{H}} z_{qk} e^{ik\omega t}\right) \tag{1.71}$$

where spatial DOFs are indexed by q and temporal DOFs are indexed by k. Thus the ordering of generalized space/time complex DOFs

$$\{\mathcal{Z}_{qk}\} = \left\{ \begin{array}{c} z_{q0} \\ z_{q1} \\ z_{q2} \\ \vdots \end{array} \right\}_{n_q.n_h \times 1} \tag{1.72}$$

The harmonic function is then complex,

$$H_{lj} = e^{il\omega t_j} \tag{1.73}$$

The usual harmonic balance using Fourier coefficients, one has the equivalence

$$z_{qk} = z_{cqk} - iz_{sqk} \tag{1.74}$$

as $e^{ik\omega t} = cos(k\omega t) + isin(k\omega t)$, the equivalence between (1.71) and (1.58) is indeed

$$\Re(z_{qk})cos(k\omega t) - \Im(z_{qk})sin(k\omega t) = z_{cqk}cos(k\omega t) + z_{sqk}sin(k\omega t) \tag{1.75}$$

For each harmonic k the mean equilibrium over a period (1.65) leads to an equation of the form

$$\left(-(k\omega)^2 M + ik\omega C + K\right) \{z_{qk}\} + \left(F_{nl}(q, \dot{q}, q_{nl}, \omega, t_i) - F_{ext}(\omega, t_i)\right) [H_{tk}] = 0 \tag{1.76}$$

where

$$F_{nl}(Z, \omega) = -\, [b]\, s_{nl} \tag{1.77}$$

Since b is a constant matrix, the harmonic contribution of a non-linear stress can be computed at the gauss point level using complex notation

$$s_k = s_{ck} - is_{sk} = \mathcal{F}_k^{-1}\left(s_{nl}(Z, \omega, t_i)\right) = \sum_{i=1}^{N} \left(\cos(k\omega t_i) + i\sin(k\omega t_i)\right)\left(s_{nl}(Z, \omega, t_i)\right) \tag{1.78}$$

For a linear spring with a loss factor and harmonic strains u_k

$$s_k = \mathcal{F}_k^{-1}\left(K(1 + i\eta)u(Z, \omega, t)\right) = K(1 + i\eta)u_k \tag{1.79}$$

Assuming a scalar stress relation, the non-linear harmonic problem (1.76) is thus equivalent to a linear harmonic problem

$$\left(-\omega^2 M + i\omega C + K + [b]\, K_{eq,k}\, [c]\right) \{\mathcal{Z}_k\} - F_{ext}(\omega) = 0 \tag{1.80}$$

with the equivalent complex stiffness given for each Gauss point by

$$K_{eq} = \frac{\mathcal{F}_k^{-1}\left(s_{nl}(Z, \omega, t_i)\right)}{u_k} \tag{1.81}$$

This expression of the problem is the basis for fixed point solvers, where a starting $K_{eq,k}$ is introduced.

1.6.4 Inter-harmonic coupling discussion

Harmonic balance Jacobian estimation based on local non-linear physics is rarely discussed in the literature. Possible for a given harmonic to estimate an equivalent local linear constitutive law. This notion comes close to quasi-Newton methods in transient simulations, that estimate Jacobians with fixed operators based on the non-linear constitutive laws.
The harmonic balance framework adds some complexity as the transient response is decomposed in the time domain. It seems however possible to assess Jacobian relevant topologies using Taylor series expansion when the non-linear forces can be expressed in a functional way (coherent with the existence of a constitutive law).

$$[J_{nl}] = \frac{\partial f_{nlj}}{\partial z_{hqk}} \tag{1.82}$$

$$f_{nlj} \simeq k_1\epsilon + k_2\epsilon^2 + k_3\epsilon^3 + \cdots \tag{1.83}$$

$$\epsilon \simeq \epsilon_{cq0} + \epsilon_{cq1}\cos(\omega t) + \epsilon_{sq1}\sin(\omega t) + \epsilon_{cq2}\cos(2\omega t) + \epsilon_{sq2}\sin(2\omega t) + \epsilon_{cq3}\cos(3\omega t) + \epsilon_{sq3}\sin(3\omega t) + \cdots \tag{1.84}$$

First order terms induced by the linear and cubic constraint term respectively induced by $\epsilon = \epsilon + \delta_{cq1}\cos(\omega t)$ and $\epsilon = \epsilon + \delta_{sq1}\sin(\omega t)$

$$\left\{ \begin{array}{l} \left(k_1 - 9\epsilon_{cq1}^3 k_3\right)\delta_{cq1}\cos(\omega t) + \frac{3}{4}\epsilon_{cq1}^2 k_3\delta_{cq1}\cos(3\omega t) \\ \left(k_1 + 9\epsilon_{sq1}^3 k_3\right)\delta_{sq1}\sin(\omega t) - \frac{3}{4}\epsilon_{sq1}^2 k_3\delta_{sq1}\sin(3\omega t) \end{array} \right. \tag{1.85}$$

First order terms induced by the linear and cubic constraint term respectively induced by $\epsilon = \epsilon + \delta_{cq3}\cos(3\omega t)$ and $\epsilon = \epsilon + \delta_{sq3}\sin(3\omega t)$

$$\left\{ \begin{array}{l} \left(k_1 - \frac{9}{4}\epsilon_{cq3}^2 k_3\right)\delta_{cq3}\cos(3\omega t) + \frac{3}{4}\epsilon_{cq3}^2 k_3\delta_{cq3}\cos(9\omega t) \\ \left(k_1 + \frac{9}{4}\epsilon_{sq3}^2 k_3\right)\delta_{sq3}\sin(3\omega t) - \frac{3}{4}\epsilon_{sq3}^2 k_3\delta_{sq3}\sin(9\omega t) \end{array} \right. \tag{1.86}$$

First order terms induced by the linear and quadratic constraint term respectively induced by $\epsilon = \epsilon + \delta_{cq1}\cos(\omega t)$ and $\epsilon = \epsilon + \delta_{sq1}\sin(\omega t)$

$$\left\{ \begin{array}{l} \epsilon_{cq1}k_2\delta_{cq1} + k_1\delta_{cq1}\cos(\omega t) + \epsilon_{cq1}k_2\delta_{cq1}\cos(2\omega t) \\ \epsilon_{sq1}k_2\delta_{sq1} + k_1\delta_{sq1}\sin(\omega t) - \epsilon_{sq1}k_2\delta_{sq1}\cos(2\omega t) \end{array} \right. \tag{1.87}$$

First order terms induced by the linear and quadratic constraint term respectively induced by $\epsilon = \epsilon + \delta_{cq3}\cos(2\omega t)$ and $\epsilon = \epsilon + \delta_{sq2}\sin(2\omega t)$

$$\left\{ \begin{array}{l} \epsilon_{cq2}k_2\delta_{cq2} + k_1\delta_{cq2}\cos(2\omega t) + \epsilon_{cq2}k_2\delta_{cq2}\cos(4\omega t) \\ \epsilon_{sq2}k_2\delta_{sq2} + k_1\delta_{sq2}\sin(2\omega t) - \epsilon_{sq2}k_2\delta_{sq2}\cos(4\omega t) \end{array} \right. \tag{1.88}$$

1.6.5 Load definition

The SDT definition of loads `DofLoad` or enforced displacement `DofSet` use the formalism of input shape matrices. Thus the time dependence of the load is given by

$$\{F(t)\} = [b]\{u(t)\} \tag{1.89}$$

where $[b]_{N\times NS}$ describes the spatial content and the harmonic content is fully contained in $\{u(t)\}$. u is often scalar but can be a vector if multiple loads are combined. For each u component j, a two dimensional curve is defined giving

$$\{u_j(t)\} = u_{c0j}(\omega) + \sum_{k\in\mathcal{H}} u_{skj}(\omega)sin(k\omega t) + u_{ckj}(\omega)cos(k\omega t) \tag{1.90}$$

The harmonic load frequency dependence is then defined for each load vector b_j by scalar coefficients associated to each harmonic

$$\{u_j(\omega)\} = \left\{\begin{array}{c} u_{c0j}(\omega) \\ u_{s1j}(\omega) \\ u_{c1j}(\omega) \\ \vdots \end{array}\right\} \tag{1.91}$$

These terms are declared by the field `.curve` defined in the `Load` structure.

- The field can be omitted or left empty. It then assumed that the force is a constant value associated to the `c1` harmonic.
- The field can be a single curve, producing a scalar amplitude value per harmonic. The same amplitude is then applied to all loads.
- The field can be a cell array of curves, each curve being associated to a column of `Load.def`. The result of each curve is then coherent with the harmonics declared in the curve.

A given curve entry will provide the frequency dependency of a given Load vector for a specified set of harmonic shapes.
It can be defined using a *Tabular form*, or a *Functional* form by a structure coherent with `sdtweb curve` formats, with fields

- `.X` A `1x2` cell-array.
  - `.X{1}`The first cell array provides the base pulsation vector that will provide the linear interpolation coefficients. This can be left empty for functional definitions.
  - `.X{2}` The second cell is a column cell-array providing the harmonic shape labels to whom the load is applied.

- `.Y` the field providing the amplitude
  - Tabular form: a matrix with as many lines a in field `.X{1}` and as many columns a the number of harmonics provided in field `.X{2}`.
  - Functional form using MATLAB anonymous function handles. A structure with fields
    * `.anonymous` Provides the inline function. The anonymous function header is set by default if omitted, `@(Zf,w)`. The inline can access the curve structure `Zf` whose fields will contain the fields declared in `curve.Param`, and the current pulsation `w`.
    * `.csv` A parameter declaration string under `cingui` `ParamEdit` format. This declares the parameters to be used, with a default value, their type and a possible brief explaination.
    * `.Param` The current parameters. `.Param` can be
      · a string defining the parameters declared in the `.csv` by `par1=val1 par2=val2 ...`
      · a structure with fields corresponding exactly to the declared parameters `struct('par1', val1, 'par2', val2)`.

    Any omitted parameter will be set to its default declared in the csv. Lack of default values would then results in an error at the function execution.
    * `.tex` a string providing a `tex` format of the formula used in `.anonymous`. Lazy declaration can be done by providing a string using `w` instead of the structure format.

```
% tabular formulation
C1=struct('X',{{1, ... % Frequencies can be a column vector if varying
    {'c0';'c1'}}}, ...  % harmonic labels, see hdof
    'Xlab',{{'Frequency','harm'}}, ... % Frequency Hz or Pulsation rad/s
    'Y',[.1 50]);  %
% Functional formulation
C2=struct('X',{{[],{'s1'}}},...
'Y',{{struct('anonymous','Zf.k1*w^2',...
'Param','k1=1e-2',....
'csv','k1(1#%g#"")',...
'tex','k_1 \omega^2')}});
% Lazy anonymous
C3=struct('X',{{[],{'c0';'c2'}}},'Y',{{'10*sqrt(w)','w^2'}});
```

Internally this definition is transformed to use a command matrix $[b]_{Nhdof \times Nhload}$, with N_{hdof} the number of harmonic DOF defined in field `.hdof` and N_{hload} the number of harmonic loading. One physical load is replicated by the number of harmonics specified in the curve field `.X` input to allow distinct amplitudes per harmonic.

At a given pulsation, a vector $u(\omega)_{Nhload\times 1}$ is generated by parsing the curve inputs,

$$\{u(\omega)\} = \left\{ \begin{array}{c} \vdots \\ \{u_j(\omega)\} \\ \vdots \end{array} \right\} \tag{1.92}$$

so that the total external harmonic load vector is expressed as

$$\{Z_f\}_{Nhdof\times 1} = [b]\,\{u(\omega)\} \tag{1.93}$$

1.7 Data structures for HBM solvers

1.7.1 Model, superelement

The `model` structure containing in particular

- `model.K` list of matrices involved in the computation
- `model.Klab` list of labels describing each matrix
- `model.Opt(2,:)` list of labels describing each matrix
- `model.NL` stack of non-linearities.

This section describes a subset of superelement specifications described in more details in `sdtweb('secms')`. The structure is a standard OpenFEM model structure with additional fields described below.

`Opt`

Options characterizing the type of superelement as follows:

`Opt(1,1)`	1 classical superelements.
`Opt(2,:)`	matrix types for the superelement matrices. Each non zero value on the second row of `Opt` specifies a matrix stored in the field `K{i}` (where `i` is the column number). The value of `Opt(2,i)` indicates the matrix type of `K{i}`. `1` stiffness, `2` mass, `3` viscous damping, `4` hysteretic damping.

`Node`

Nominal node matrix. Contains the nodes used by the unique superelement. The only restriction in comparison to a standard model `Node` matrix is that it must be sorted by `NodeId` so that the last node has the largest `NodeId`.

K{i},Klab{i},DOF

Superelement matrices. The presence and type of these matrices is declared in the Opt field (see above) and should be associated with a label giving the meaning of each matrix.
All matrices must be consistent with the .DOF field which is given in internal node numbering.

Elt, Node, il, pl

Initial model retrieval for unique superelements. Elt field contains the initial model description matrix which allows the construction of a detailed visualization as well as post-processing operations. .Node contains the nodes used by this model. The .pl and .il fields store material and element properties for the initial model.
Once the matrices built, SE.Elt may be replaced by a display mesh if appropriate.

TR

TR field contains the definition of a possible projection on a reduction basis. This information is stored in a structure array with fields

- .DOF is the model active DOF vector.
- .def is the projection matrix. There is as many columns as DOFs in the reduced basis (stored in the DOF field of the superelement structure array), and as many row as active DOFs (stored in TR.DOF).
- .hdof, when appropriate, gives a list of DOF labels associated with columns of TR.def
- .data, when appropriate, gives a list frequencies associated with columns of TR.def
- .KeptDOF can be used to specify master DOFs not included TR.def but that should still be used for display of the superelement.

1.7.2 Non-linearity definition NLdata

Initialization of non-linear behavior in a cbush element group is performed with the following NLdata formats
Definition with custom functions. The NLdata property must contains the following fields

- type='nl_inout' to let hbm_solve InitHBM build the needed observation matrix
- Fu='@UserFun' references a user function computing the non-linear force with the prototype call described in section **??** . Note that in instances of deployed MATLAB generated with the MATLAB compiler, all custom functions must be defined a priori. And only anonymous functions may be created.

- `adofi` to declare internal states, in coherence with field `.MatTyp` below. This field can be omitted of left empty if no such feature is used. Internal states are defined independently for each observation line used in the non-linearity. *e.g.* For a `cbush` six directions are available relative to the 3 translations and 3 rotations that can be observed. `.adofi` is then a line cell array of length the number of observations. Each cell defines a number of internal states associated to the corresponding observation index by providing a column vector with as many lines as internal states used each containing the DOF extension .99. The cell is left empty if no internal state is declared for a particular direction. `NLdata.adofi={[];[];[];[];[];.99}` will add an internal state to the 6^{th} observation of the non-linearity.
- `.MatTyp` : declares the time derivative of the signal associated to each internal DOF. This corresponds to matrix definitions in the Jacobian. Thus `MatType=1` stiffness corresponds to a displacement DOF, `2` mass an acceleration DOF, `3` viscous damping a velocity DOF.
- `.isens` may be used to select a partial list of strains normally computed. For example to only keep translations of a `cbush` use `.isens=[1 2 3]`.

1.7.3 NL structure non-linearity representation during HBM solve

`NL` structures describing each non-linearity

- `NL.c` standard observation matrix for the observed motion used to express load. Initially in physical coordinates and transformed to harmonic observation in `Build_c_unl_k`.
- `NL.b` command matrix to reapply harmonic loads on the proper DOFs.
- `NL.Fu` cell array of in-line functions.
- `NL.type` string containing the non-linearity type
- `NL.c0,NL.b0` observation/command of non-linear strain in physical space only.

1.7.4 Solver options definition

```
out=struct('Method','hbm_solve',...
 'Opt',[0 9 1 .01 .4 0 1],... [adapt Nhmax nu fmin fmax UNU fvar]
 'SaveFreq',[0 .01 1e3 ],... [stra (full/block) fstep nFpoints]
 'RelTol',1e-9,'MaxIter',12,...
 'Rayleigh',[0 0],...
 'NeedUNL',[0 0],...
 'AssembleCall',hbm_solve('AssembleCall'),...
 'JacobianUpdate',pmat(zeros(1)),...
```

```
'iterHBM','@iterHBM',...
'initHBM','@initHBM',...
'resHBM','@resHBM',...
'abscHBM','@abscHBM',...
'FinalCleanupFcn','hbm_solve(''fe_timeCleanup-cf-1'');',...
...'dSOpt',[ 2 1 0 .01 .01 .1  7 .3 1.5 6 12  ]); % [  Ldeg absc step dsfix dsmin dsma
'dSOpt',[1 .01 .01 .1  7 .3 1.5  2 ]); %step(lin/lagrange) dsfix dsmin dsmax iopt bmin
```

- `opt.Method` (`'hbm_solve'`) Provides the method used by the solver. The default is `hbm_solve` for a frequency scan.
- `opt.AssembleCall` (= `hbm_solve('AssembleCall'')`) Provides the call to perform an assembly performing initialization specific to the HBM module. This field is usually left by default, using the output of command `hbm_solveAssembleCall`.
- `opt.Jacobian` (= `''`) Provides a callback to compute the Jacobian used for the solver resolutions. The default procedures peforms Jacobian computations during residue computation so that this field is empty by default.
- `opt.JacobianUpdate` (= `pmat(ones(1))`) A `pmat` indicator controlling the Jacobian updating procedure. This can be set to `0` to ask not to update the current Jacobian, or to `1` to trigger an update. This value can be modified by the solver if automated Jacobian update schemes are used (see `opt.juit`).
- `opt.initHBM` (=`'@initHBM'`) Provides a function handle called for data initialization before solve. The internal method `initHBM` is used by default, this field is thus initialized with the internal handle name of the `hbm_solve` methods.
- `opt.abscHBM` (=`'@abscHBM`') Provides a callback to perform curvilinear frequency predictions based on a buffer of response. The internal method `abscHBM` is used by default (provides linear or Lagrange polynomial interpolation), this field is thus initialized with the internal handle name of the `hbm_solve` methods.
- `opt.iterHBM` (=`'@iterHBM`') Provides a callback to perform equilibrium iteration loops at a given point. The internal method `iterHBM` is used by default, this field is thus initialized with the internal handle name of the `hbm_solve` methods.
- `opt.resHBM` (=`'@resHBM`') Provides a callback to compute the equilibrium residue of a given HBM state. The internal method `resHBM` is used by default (also provides Jacobian computation), this field is thus initialized with the internal handle name of the `hbm_solve` methods.
- `opt.Opt` (= `[adapt Nhmax nu fmin fmax UNU fvar]`)

- adapt (= 0) Unused, must be left to 0.
- Nhmax (= 9) Defines the maximum number of harmonics considered, drives the transient buffer size.
- nu (= 1) Defines the harmonic factor $\omega = k/\nu$.
- fstart (= .01) Defines the starting frequency for scanning.
- fend (= .4) Defines the end frequency for scanning.
- UNU (= 0) Unused value left to zero for OpenFEM retro-compatibility.
- fvar (= 1) Declares the frequency representation, either fixed (fvar = 0) or unknown (fvar = 1). If set to 1 the harmonics vector Z is augmented with the pulsation, corresponding to harmonic DOF 1.99,'freq'.

- opt.Rayleigh (= [alpha beta]) Provides Rayleigh damping values applied to system matrices, defining $C = \alpha M + \beta K$.

- opt.juit (= -Inf) Defines an automated Jacobian update strategy used in resHBM, the Jacobian is then updated only after juit iterations. The default value set to -Inf asks for an update at each step.

- opt.dSOpt (= [stra dsfix dsmin dsmax iopt bmin bmax Ldeg]) Defines the curvilinear frequency prediction for continuation techniques, used by abscHBM.

  - stra (= 1) defines the increment strategy, either fixed (0) or with Lagrange polynomials (1).
  - fix (= .01) defines the fixed pulsation step , used for fixed strategy and to initialize other ones.
  - min (= .01) defines the minimum authorized pulsation step.
  - max (= .1) defines the maximum authorized pulsation step.
  - iopt (= 7) defines an optimal iteration number indicator. Pulsation frequency continuations will be modulated to target iopt iteration for convergence.
  - bmin (= .3) defines the minimal modulation factor applied to reach iopt iterations for convergence.
  - bmax (= 1.5) defines the maximal modulation factor applied to reach iopt iterations for convergence.
  - Ldeg (= 2) defines the degree of the Lagrange polynomial extrapolation.

- opt.RelTol (= 1e-9) defines the tolerance to consider the HBM equilibrium is attained.

- opt.MaxIter (= 12) defines the maximum number of iterations allowed before stopping the loop.

- `opt.SaveFreq` (`= [stra fstep nFpoints]`) defines the output storage strategy.
  - `stra` (`= 0`) defines the storage strategy. `0` defines a direct output saving, saving as much as `nFpoints` results every time the module frequency change is greater than `fstep` since the last save.
  - `fstep` (`= .01`) defines the frequency module evolution step triggering a save.
  - `nFpoints` (`= 1e3`) defines the total number of results saved in the output.
- `opt.NeedUNL` (`= [0 0]`) asks to save `unl` (if opt.NeedUNL(1)==1) and/or `vnl` (if NL.NeedUNL(2)== observations.
- `opt.FinalCleanupFcn` Provides the call to perform output cleanup after resolution. This field is usually left by default, calling `hbm_solve('fe_timeCleanup''`. Token `-cf-1` directly stores the result in the GUI.

1.7.5 Option structure during HBM solve

Inside the solver loop, developers may want to access a number of parameters described below. The internal structure during time solves is described in `sdtweb('nldata#nlformtime')`;

- `opt.N` number of samples for time signal
- `opt.A` A matrix (1.66) (this matrix gets overwritten during iterations).
- `opt.Hkt` unit evolution of a given harmonic.(1.61)
- `opt.dHkt` time derivative of unit evolution of a given harmonic.
- `opt.hdof` two column matrix giving for each DOF the physical DOF and the time variation index.
- `opt.harm` internal field of retained harmonics.

1.7.6 Harmonic result structure

is the data structure used to store SDT-HBM results. It is a variation of the `curve` format. With the first dimension containing harmonic DOFs, the second frequencies and the last amplitudes.

```
sdtweb hbm_solve('outputinithbm')
 out=struct('Y',zeros(r1),...
  'X',{[{hdof}, freq,amp]},...
  'Xlab',{'Hdof','Freq','Amp'},...
  'hdof',{opt.hdof},'DOF',Case.DOF,...
  'harm',opt.harm,'idof',opt.idof,'ihdof',opt.ihdof,'cur',zeros(1,max(3,length(r1)+1)))
```

2

Tutorial

Contents

2.1 Installation

Steps of an installation are

- Install SDT.
  - The current starting point is SDT 6.8 beta which can be dowloaded from `http://www.sdtools.com/distrib/beta/sdtcur_dis.p`. To obtain a SDT license key, you should then use the procedure at `http://www.sdtools.com/faq/Release.html`.
  - To save multiple SDT installations, see `http://www.sdtools.com/faq/Release.html#multi`
  - You must have write permission in the SDT directory so that it can be patched by yourself. If this is not the case you should install SDT somewhere in your own directories. For example use `target='c:/sdtdata/hbm/sdt.cur'` to avoid install to the traditionnal directory `matlabroot/toolbox/sdt`, see `http://www.sdtools.com/faq/Release.html#multi`.
  - For multi-boot systems (windows, linux), use a single SDT (see item above). This will avoid the need to install patches on both installations.
- For the target SDT **a patch is always needed**. Patches to SDT are files named `hbm_patch_disp.p`. They can be obtained using a call of the form

```
hbm_utils('DistribGetPatch') % Install the patch
hbm_utils('DistribCheck')    % Run basic check of versions
```

  - The command only downloads the patch, you are then supposed to click on the link `cd('d:/del/scratch');hbm_patch_dis;rehash toolboxreset % do` so that the patch is installed. The two step procedure is needed to give you a chance that the location of the patch install is correct.
  - You must have write permission in the SDT directory, see first item of install.
- a copy of the SDT-HBM code. Two cases are possible
  - a deployement version obtained from SDTools as a crypted 7z file, this requires the existence of a `license.txt` file in your base SDT directory.
  - a SVN version obtained by a checkout on URL `http://support.sdtools.com/svn/hbm/trunk`. For details on SVN clients to do a checkout, contact SDTools. It is expected that your rename your local copy of the `trunk` directory `HBM`. This will then be the base of your SDT-HBM directory structure.

The SDT-HBM directory structure is

- `hbm/m` contains all the Matlab files needed to run SDT-HBM.
- `hbm/help` contains the documentation, see `hbm_utils Help` to update.
- `sdt.cur` classical location for SDT installation associated with SDT HBM.
- `hbm/test` contains the files needed for testing. This is used for `t_hbm` log.
- `hbm/tex` the root file is `hbm.tex`. It contains all the includes for other documentation files. Figures are stored as `.pdf` or `.png` in directory `plots` subdirectory, see `hbm_utils Latex` to recompile. Documention contributions are welcome.

2.2 Frequency domain test cases

2.2.1 Example lists

- `d_hbm('TestDuffing2dof')` spring mass duffing example detailed in section 2.3.1
- `model=d_hbm('TestDofSet1')` provides a test case with enforced displacement.
- `d_hbm('TestBeamNL')` is a simple beam with a rotation spring. This was analyzed in detail in [?].
- `d_hbm('TestBeamVNL')` is similar but implements the rotation spring as a single volume element. This is used to validate the implementation of volume non-linearities described section 1.1.4 .
- `d_hbm('TestSPlate')` illustrates a plate on non-linear supports.
- t_hbm TestLapJoint Bj2 (provided by AGI as part of project CLIMA)

2.2.2 Spring mass examples

The 2DOF duffing model provided by AGI is implemented in `d_hbm('TestDuffing2dof')`.

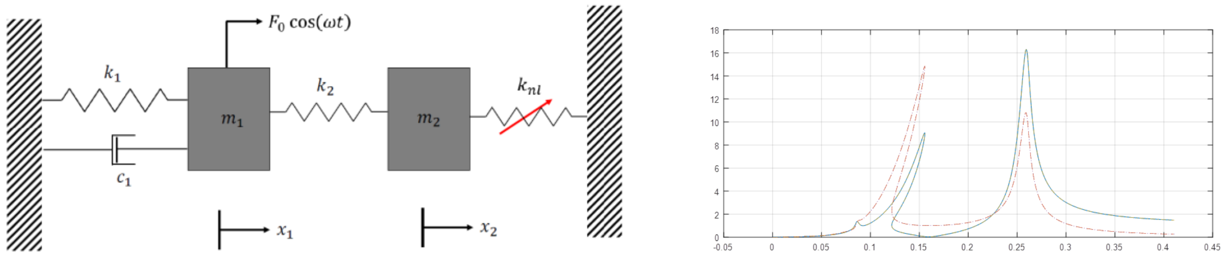


Figure 2.1: Two DOF duffing oscillator with cubic non-linearity

```
sdtweb d_hbm('TestDuffing2Dof'); % Open source code of example
[mo1,opt,Z,XF]=d_hbm('TestDuffing2dof'); % Run and display
```

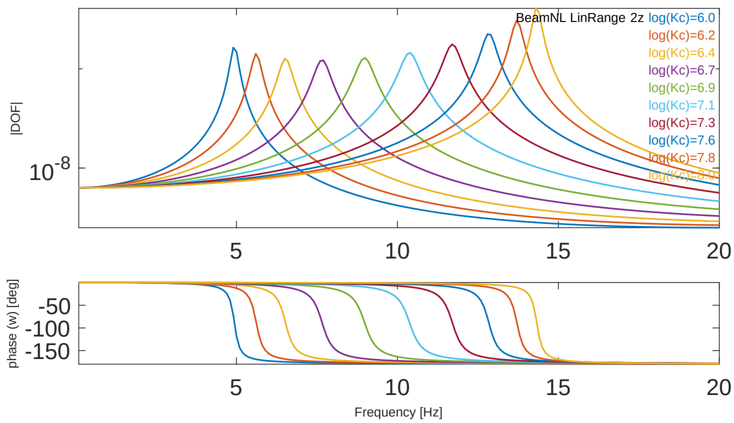
2.2.3 Beam problem with local non-linearity

```
sdtweb t_hbm beamnl
```

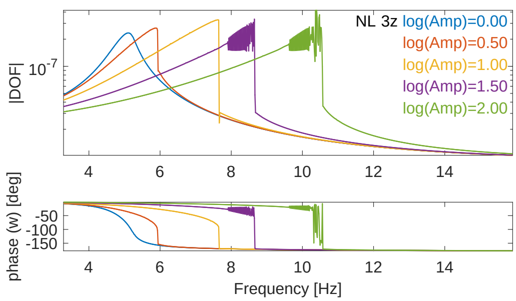
As a first example `clima16('BeamNLKrange')`, one seeks to demonstrate the sensitivity to a variable stiffness. The non-linear stress strain relation is defined by

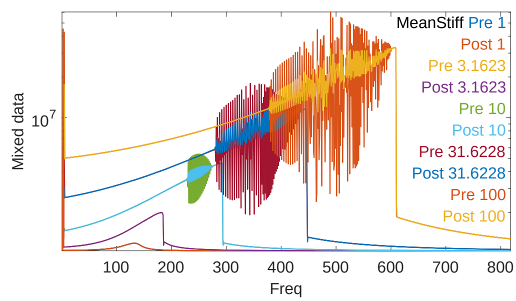
```
Fu=@(NL,fc,model,u,v,a,opt,Case,RO)kcur.*(NL.unl-loss*NL.vnl/opt.w);
```

which combines a constant stiffness `kcur` and a loss factor defined in the time domain using a frequency dependent velocity contribution. Taking $q = cos(\omega t)$, one assumes the equality of the complex stiffness and viscous damping forms in the stress/strain relationship $s = Re\left(k(1+i\eta)e^{i\omega t}\right) = Re\left((k + i\omega c e^{i\omega t})\right)$ which leads to $c = -\eta/\omega$. In the resulting frequency responses below, one clearly sees a frequency response having a transition from a lower frequency at 4 Hz for $kcur = 10e6$ (in model units) and an upper frequency at 15 Hz for $kcur = 1e8$. The figure also clearly shows that damping decreases close to the limits as expected (see [?] for details).



As a second example `clima16('BeamNLARange')`, one seeks to illustrate the amplitude dependence obtained for a stiffening spring.





2.2.4 Lap joint problem with contact

Current simple test is found in `t_contact('LapJzt')`. Variants include one or 3 bolts. Different strategies to generate the response.
The current test `clima16('LJEB')`.

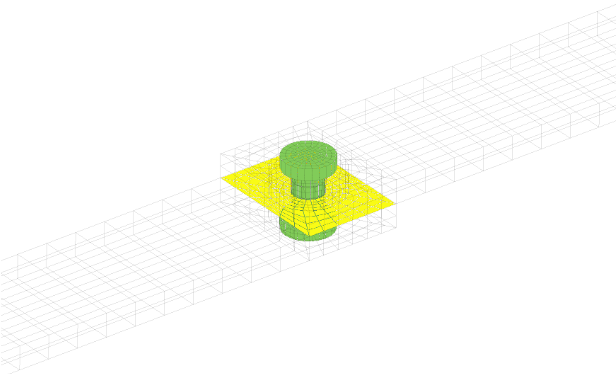


Figure 2.2: Lap joint with contact surface

2.2.5 Hyperelastic bushing

This example is a functional demonstration of capabilities associated with hyper-viscoelastic behavior. It is based on the RotDamper example.

```
[mo1,hopt]=d_hbm('TestBeamVNL');
```

2.3 Time domain test cases

- sdtweb('_eval','d_fetime.m#BumpStop') simple mass on spring with bumpstop non-linearity.

2.3.1 Single mass test of various non-linearities

Single mass test of various non-linearities t_nlspring ModalNewmark. Supported examples are

- Maxwell viscoelastic spring,
- tabular stiffness,
- Dahl model with constant normal force.

```
% Sample example with tabular stiffness and output spectrogram
 sdtweb('_eval','d_fetime.m#Maxwell')
 mdl=stack_set(mdl,'info','DefaultZeta',@(f)zeros(size(f)));

 x=[-100 -1e-4 1e-4 100]';Fu=struct('X',{{x}},'Y',x.*[10 0 0 10]');
```

```
 NLdata=struct('type','nl_inout','lab','TabK','keepLin',0,...
  'isens',3,'MatTyp',{{[3]}},'Fu',{{Fu}});
 mdl=feutil('setpro 1000',mdl,'NLdata',NLdata);
 mo2=nl_solve('ReducFree 2 10 0 -float2 -SE',mdl);% With internal DOF
opt=d_fetime('TimeOpt dt=1e-4 tend=100 ModalNewmark');
RB=struct('spec','BufTime 20 Overlap .75 fmin0 fmax60 -window hanning','ci',3);
opt=stack_set(opt,'ExitFcn','Tip', ...
  struct('FinalCleanup',{{'nl_solve','PostCdof'}},'DOF',2.03,'DoFreq',RB));
opt=stack_set(opt,'info','RangeTime',fe_range('grid',struct('A',logspace(-3,0,5))));
d2=fe_time(opt,mo2);d2=fe_def('subdef',d2,d2.data>d2.data(2));
```

2.3.2 Single mass stepped sine

The `d_hbm TestSteppedSine` example illustrates the stepped sine procedure implemented with the `ModalNewmark` solver.
See also `d_tdoe`xxx.
In such computations, it is assumed that the excitation frequency is fixed during the transient, so that the following parameters can be used the the run options

- `.Nper` number of periods for each computation
- `.NperPer` number of points per period. Alternatively time step `.dt=1/freq/NperPer` can be defined and used to set the initial value of `.NperPer`).
- `.freq` vector of frequencies for a series of stepped sine simulations.
- `.A` vector of amplitudes applied globally on the load case.

After the time computation of the target number of periods the `ExitFcn` entries in `RT.Stack` are processed by order. Typical entries would be

- `PostFirstStab` performs `.ite` time simulations without trying to check for stabilization to allow the transient to stabilize before any
- `PostConstit` extract non-linear stress/strains and possibly performs harmonic extraction with `doFreq` callback.

2.3.3 CBush with orientation

The example is detailed in

```
d_fetime('tutoCbushOrient')
```

2.3.4 Beam with non-linear rotation spring

First do a sweep

```
sdtweb('_eval','d_fetime.m#TestBeamNLRed')
```

2.3.5 Lap joint with non-linear springs

This test case is discussed with *Marco Rosatello*. See `t_bjoint('TestTime')`.
The axial behavior of the C2 connector is given by a non-linear law in tabular $F_z(e_3)$ form. This force is output as the generalized stress during time computations, while for equations actually solved one uses

$$s_3(t) = F_z(e_3(t)) - k_{Jz} e_3(t) - F_0 \tag{2.1}$$

For two directions x and y, the tangential behavior is incremented using a Dahl integration scheme

$$F_x(t+dt) = F_x(t) + \sigma(e_3(t))dt\ \dot{e}_x \left(1 - \frac{F_x(t)}{\mu F_z(e_3(t))} \mathrm{sign}\left(\dot{e}_x\right)\right)^{\alpha} \tag{2.2}$$

Since this integration does not guarantee $|F_x(t)| <= |\mu F_z(t)|$, the condition is enforced at each time step. Again there is the need to distinguish $F_x(t)$ and the tangential load which needs to account for adherence stiffness in the nominal model used to generate the modal basis serving to define DOFs for time integration. Thus

$$s_1(t) = F_x(t) - k_{Jx} e_1(t) \tag{2.3}$$

Iwan model with Dahl cells. Data is *sigma* slope at no load, α shape parameter, μF_z normal load.

$$F_x(t+dt) = F_x(t) + \sigma dt\ \dot{e}_x \left(1 - \frac{F_x(t)}{\mu F_z} \mathrm{sign}\left(\dot{e}_x\right)\right)^{\alpha} \tag{2.4}$$

2.3.6 Parametric experiments in time

`nl_solve` handles experiments associated with a series of time computations defined using the `fe_range` format. Typical applications are frequency and amplitude stepping experiments associated with HBM testing. For examples see
The principle of the experiment is that at each design point parameters are first changed before starting a time computation followed by postprocessing steps. The expression for changing the parameter can be given manually as in the first `d_fetime` `doTimeRange` example where

```
R1.param.Zener_c1.SetFcn='NL.opt(10:11)=[1 -1]*10/500*val(j1);';
```

changes the constants in `NL.opt(10:11)`.
Other predefined set functions are

- `fin_coef` do a stepped scaling coefficient on the time step. The Jacobian is updated but the loading profile is kept constant. This is typical of harmonic testing.
- `A,A0` can be used to
- `m_coef` adjust density for quasi-static testing.

xxx

2.4 Elastic representation as superelements

2.4.1 Generic representations in SDT

For interfacing with external finite element software the well documented superelement formalism is used. This formalism is largely used by Multibody Dynamic Software (Simpack, Adams, Excite, ...) and thus widely documented.
A superelement representation of the model is of the form

$$\left[M_R s^2 + C_R s + K_R + iD_R\right]\{q_R(s)\} = [b_R]\{u(s)\} \tag{2.5}$$

In general, the reduction is performed so that the DOFs retained $\{q_R\}$ are related to the original DOFs of a larger model by a Rayleigh Ritz reduction basis T using

$$\{q\}_N = [T]_{N\times NR}\{q_R(s)\}_{NR} \tag{2.6}$$

This representation is fairly standard. The data structure representation within SDT is described in section 1.7.1 . SDT/FEMLink supports import from various FEM codes and more details are given in section 2.4.2 for NASTRAN and section **??** for Abaqus.
For Craig-Bampton type reduction (enforced displacement on a set of interface DOF I and fixed interface modes on other DOF C), the reduction basis has the form

$$[T] = \left[T_I \quad T_C\right] = \left[\begin{array}{ccc} I & 0 & 0 \\ -K_{CC}^{-1}K_{CI} & [\phi_C]_{1:NM} & \left[K_{CC}^{-1}[b_{res}]\right]_\perp \end{array}\right] \tag{2.7}$$

which verifies the constraints on basis columns that $T_{II} = I$ and $T_{IC} = 0$.
For the free mode variant (McNeal) (see `sdtweb('fe_reduc','free')`), the form is

$$[T_C] = \left[\begin{array}{cc} [\phi]_{1:NM} & \left[K_{Flex}^{-1}[b_{res}]\right]_\perp \end{array}\right] \tag{2.8}$$

with no T_I columns.
The observation formalism of SDT which is applicable to both test/analysis sensors (see `sdtweb('sensor')`) and strain observations used for non-linearities .

$$\{y\} = [C]\{q\} \tag{2.9}$$

Standard notions that have an equivalent in other code are

- q_I interface DOF are directly comparable in SDT and other software when using a `DofSet` entry that contains an identity enforced motion matrix on a set of DOF `idof`, typically known as MASTER degree of freedom. That can be initialized with a call of the form `model=fe_case(model, 'DofSet', 'In',struct('DOF',idof, 'def', speye(length(idof))))`
- T_C columns may correspond to generalized or internal DOFs. External software will often require that a DOF number be associated to these interfaces. xxx
- b_{res} the independent vectors used to generate residual loads are found as columns of a `DofLoad` entry. For point loads on a set of DOF `adof` the entry would be `struct('DOF',adof, 'def',speye(length(adof)))`.
- $[c]$ observation matrices do not exist in other environments. The strategy usually retained for export is to add additional nodes of a set `ObsNode` to correspond to the observations of interest and define the observation equation using a multiple point constraint. This is achieved by modifying the model using `fe_sens('MeshSensAsMPC'`.
- `rigid` assuming that the 3D motion of all nodes in the set depend rigidly on the center node motion. Can be used to define motion of sensor nodes. This tends to over-stiffen the area of connected nodes.
- `rbe3` assuming that the center node moves as the mean motion of the set of nodes is often considered to observe motion of nodes. The case dependent observations of SDT are more general, but may correspond to `rbe3`.

Implementations of these are discussed for Abaqus **??**, NASTRAN 2.4.3, ANSYS 2.4.4.

2.4.2 NASTRAN cards used for sensors/non-linearities

The NASTRAN equivalent of superelement notions discussed in section 2.4.1 are

- q_I interface DOF of are defined in NASTRAN using `Bset` cards. These are stored in SDT as a `DofSet` entry to the model.
- b_{res} the independent vectors used to generate residual loads and lead to additional shapes using the residual vector procedures of NASTRAN
  - point loads simply declared using the `USET,U6` card

  - relative loads simply can obtained by declaring a `CDAMP` element that generates a relative viscous load between its two nodes.
  - $[b]$ is defined by an `DAREA` real loading and possibly `DPHASE` definition. It should be noted that in SDT, it is strongly advised to define the phase using the input, since a complex input shape matrix has no sense in the time domain. The input is defined using a `RLOAD2` $B(f)e^{i\phi(f)+\theta-2\pi f\tau}$ or `RLOAD1` $(C(f)+iD(f))e^{i\theta-2\pi f\tau}$

- T_C columns are associated to scalar DOFs called `QSET`. These require the definition of a `QSET` card (to declare existing DOFs), `SPOINT` grids (to have node numbers to support these QSET DOFs). Note also that the `SPOINT` numbers should be distinct from other `NodeId`. The number of modes defined in the EIRGL card should be lower than the the number of SPOINT and the QSET card.

- $y = [c]\{q\}$ observation. Does not exist in NASTRAN documentation, but implemented exporting SPOINT for each observation component and an MPC for each row of the observation matrix. This is achieved by modifying the model using `fe_sens('MeshSensAsMPC'` prior to export.

- `rigid` is known as `RBE2` in NASTRAN.

- `rbe3` is known as `RBE3` in NASTRAN.

Laws without internal states are similar to `PGAP` and import will be implemented in the future.

2.4.3 ABAQUS cards used for sensors/non-linearities

The Abaqus equivalent of superelement notions discussed in section 2.4.1 are

- q_I interface DOF xxx

- b_{res} the independent vectors used to generate residual loads are written as independent load cases using xxx SeWriteBres

```
*LOAD CASE, NAME=LC000001
*CLOAD, OP=MOD
1001, 1, 1
*END LOAD CASE
```

- xxxGV resvec,

- `*EQUATION` : equivalent the SDT MPC definition with a direct constraint matrix declaration.

- `*KINEMATIC COUPLING` : equivalent of SDT `rigid` connections where the spring is connected to a master node with 6 DOF which enforce motion of a number of slave DOFs.

- `*DISTRIBUTING COUPLING` equivalent of SDT RBE3 : flexible connection where the spring is is connected to a slave node with 3 or DOF which depend from a set of master nodes.
- `*COUPLING` : specific surface based definition, followed by either a `*KINEMATIC` card for rigid or `*DISTRIBUTING` card for RBE3 formulations.
- `*MPC` : node based definition with type `BEAM` to constraint 6 DOF per node or type `PIN` to constraint the 3 translations only.
- `*CONNECTOR` : connectors provide advanced structural kinematics, type `BEAM` without elasticity definition provides a rigid connection (linearized in SDT).

2.4.4 ANSYS cards used for sensors/non-linearities

For spring representations of volumes or surfaces, a first common approach is to use so called rigid elements. ANSYS supports

- CE, CERIG, MPC184, RBE 2 : rigid connections where the spring is connected to a master node with 6 DOF which enforce motion of a number of slave DOFs.
- `TARGE 170+CONTA 173, TARGE 170+CONTA 174`

2.4.5 NASTRAN Craig-Bampton example

The superelement generation by NASTRAN is saved to an `.op2` file that is automatically transformed to the SDT superelement format by FEMLink. A sample file is given in `ubeamse.dat`.

```
ASSIGN OUTPUT2='./ubeam_se.op2',UNIT=30
$
ID DFR
SOL 101
GEOMCHECK NONE
TIME 100
$
CEND
TITLE=Generic computation of mode shapes
METHOD=1
DISP(PLOT) = ALL
SPCFORCES(PLOT)=ALL
MPCFORCES(PLOT)=ALL
$ Now extract stresses on base
```

```
SET 101=1 THRU 16
STRESS(SORT)=101
$
MPC=1
SPC=1
$ RESVEC(NOINRL)= YES
EXTSEOUT(ASMBULK,EXTBULK,EXTID=100,DMIGOP2=30)
PARAM,POST,-2
PARAM,BAILOUT,-1
$
BEGIN BULK
$EIGRL,SID,V1,V2,ND,MSGLV,MAXSET,SHFSCL,NORM
EIGRL,1,,,20
$ DOF and nodes to support modal DOF
QSET1,0,1000001,THRU,1000050
SPOINT,1000001,THRU,1000050
$ Master DOF 4 base corners
BSET1,123,1,5,8,12
$
$ Residual on 3 DOF of input node 104
USET,U6,104,123
$
$ Residual associated with CAMP1 vector
CDAMP1  161     2       114     1       244     1
PDAMP*  2               1.
$
include 'ubeam_include.bdf'
ENDDATA
```

The resulting basis has the following form

$$[T] = \begin{bmatrix} I & 0 & 0 \\ -K_{CC}^{-1}K_{CI} & [\phi_C]_{1:NM} & \left[K_{CC}^{-1}\,[b_{res}]\right]_{\perp} \end{bmatrix} \tag{2.10}$$

The op2 file contains nodes and superelement definition. It is advised to read the bulk file to obtain a model containing elements and material properties.

- The interface DOF are defined in NASTRAN using`Bset` cards. These are stored in SDT as a `DofSet` entry to the model.
- `QSET` correspond to modal/generalized DOFs. These require the definition of a `QSE`T card (to declare existing DOFs), `SPOINT` grids (to have node numbers to support these QSET DOFs).

Note also that the SPOINT numbers should be distinct from other NodeId. The number of modes defined in the EIRGL card should be lower than the the number of SPOINT and the QSET card.

- Fixed interface modes ϕ_c are computed by specifying the EIRGL card.
- Residual loads b_{res} are defined as follows and lead to additional shapes using the residual vector procedures of NASTRAN
  - point loads simply declared using the USET,U6 card
  - relative loads simply obtained by declaring a CDAMP element that generates a relative viscous load between its two nodes.

2.4.6 Free mode using NASTRAN

When using a free mode computation, NASTRAN provides mechanisms to compute residual vectors, you should just insert the RESVEC=YES card. An example is given in the ubeamfr.dat file. The resulting basis has the following form

$$[T] = \left[\ [\phi]_{1:NM} \quad \left[K_{Flex}^{-1}\left[b_{res}\right]\right]_{\perp}\ \right] \tag{2.11}$$

The main mechanisms to generate residual vectors are

- Free interface modes ϕ_c are computed by specifying the EIRGL card.
- Residual loads b_res are defined as follows and lead to additional shapes using the residual vector procedures of NASTRAN
  - point loads simply declared using the USET,U6,NodeId,DofList card. Alternatively RVDOF (MSC but possibly not NX-NASTRAN) can given a list of up to four NodeId,Dof per card.
  - relative loads simply obtained by declaring a CDAMP element that generates a relative viscous load between its two nodes.

2.4.7 Storing advanced SDT options in bulk format

For upcom parameters, export is done using design variables.
SDT-NLSIM provides an harmonic definition mechanism (see hdof). Storage in NASTRAN bulk format is as follows

```
$   1  $$   2  $$   3  $$   4  $$   5  $$   6  $$   7  $$   8  $$   9  $
$ All DOFs with sin(omega t) and cos(omega t)
DTI     HDOF    1       ALL     123456  CS1     ENDREC
```

```
$ Gradual building of full list of DOFs
DTI     HDOF    1       N1      THRU    N2      123     S1      N3
        THRU    N4      1       C1      N5      123456  S1      ENDREC
```

Node numbers are first specified using `ALL` all (independent) nodes, `N1 THRU N2` a list of consecutive node numbers, N5 a single node number. Associated DOFs are then written using the CM field of RBE2 (Component numbers of the dependent degrees-of-freedom integers 1 through 6 with no embedded blanks). A third field then specifies the harmonics. `cs1` is a short cut for both $\cos(1\omega t)$ and $\sin(1\omega t)$.
The specification of target frequencies follows the normal NASTRAN format using FREQ or FREQ1 cards. Provision for a single call generating responses at multiple amplitudes (`hbm_solve AFMap .Freq` and `.Amp` fields) is specified as a DTI `HBMAmp` entry with all target amplitudes given.

```
$   1  $$   2  $$   3  $$   4  $$   5  $$   6  $$   7  $$   8  $$   9  $
EIGRL,10,,,1
$FREQ,SID,F2,F2,F3
$FREQ1,SID,F1,DF,NDF
FREQ          10   0.318      1.     3.0     4.0
RLOAD1        10       1                      1

$   1  $$   2  $$   3  $$   4  $$   5  $$   6  $$   7  $$   8  $$   9  $
DTI     HBMAmp  1       1.0     2.0     3.0     ENDREC
```

To specify loads, a number of formats are defined.

```
$   1  $$   2  $$   3  $$   4  $$   5  $$   6  $$   7  $$   8  $$   9  $
DTI     Name    1       SID     101     FORM    Amp     UN1     UN2
        Harmi   ACi     ASi     ENDREC
```

- `Name` is an arbitrary string (at most 8 characters) but should be unique and differ from internal NASTRAN tables. By default it is proposed to use strings of the form `P101` where 101 is the property number.
- `IREC` (field 3 of the DTI) is only used when considering multiple entries with the same name and should be set to 1.
- `SID` : first the string SID in field 4 then, in field 5, the property identifier (integer) which should correspond to the set identification number `SID` for which this amplitude dependence is defined.
- `Form` (selected with the string on field 7) is the form name with the following formats defined

Form		
Amp	amplitudes	$\{u(t)\} = C_0 + \sum_{k\in\mathcal{H}} S_k sin(k\omega t) + C_k cos(k\omega t)$
AmpT	Amp table	$\{u(t)\} = C_0(\omega) + \sum_{k\in\mathcal{H}} S_k(\omega) sin(k\omega t) + C_k(\omega) cos(k\omega t)$

- `Harmi` number of retained harmonic. 1 for $\cos(1\omega t)$ and $\sin(1\omega t)$.
- `ACi`, `ASi` amplitudes associated with the cosine and sine harmonic contributions. In the `AmpT` form integer numbers referring to table entries in the bulk.

2.4.8 Abaqus Craig-Bampton example

Superelement generation in Abaqus is divided in three steps.

- `*STEP, PERTURBATION//*STATIC` used to define residual vectors. Note that export of residual loads associated with non-linearities is not yet implemented in SDT.
- `*FREQUENCY, EIGEN=LANC` to compute internal modes with possibly a Craig-Bampton interface declared by a `*BOUNDARY` card.
- `*SUBSTRUCTURE GENERATE` to generate and export the superelement, use `*RETAINED NODAL DOF` associated to fixed DOF in the frequency step for a Craig Bampton reduction.

```
See SeGenResidual.inp
```

2.4.9 Abaqus McNeal example

xxx

2.4.10 ANSYS Craig-Bampton example

The cards typically used for superelement generation in ANSYS are

- `antype,substr` specify FE substructure generation in after `/SOLU`
- `cmsopt,fix,Nshapes,,,,,tcms` card generates `.sub`. Use `ans2sdt('subSE','file.sub')` to import matrices from `.sub`, restitution from `.tcms` and mesh from `.cdb` files using the same file root.
- `resvec,on` to use residual vectors in the basis
- `seopt,name,MatType,1` with `MatType=2` for mass and stiffness, and `3` for stiffness, mass, viscous damping, `name` must be defined with card `/FILENAME` before the analysis.

- `m,NodeId,all` master DOF definition repeat card for the various interface nodes. You will have to replace `all` with `UX,,UY,UZ,ROTX,ROTY,ROTZ` if the DOF is used by an element that supports multi-physics.

```
/FILENAME,ubeam_se  ! name must be the one used in SeOpt command
/PREP7
!...
! Use command F to apply loads that will define the residual vector
/SOLU

antype,substr  ! substructure analysis

CmsOpt,Fix,20,,,,,TCMS
RESVEC, ON
SeOpt,ubeamse_ans,3,1,0,,  ! 3(all matrices,2 for m and k), 1 to print

! Define list of master DOF, you cannot use ALL if the elements support multi-physics
M,1,UX,,,UY,UZ,ROTX,ROTY,ROTZ
M,5,UX,,,UY,UZ,ROTX,ROTY,ROTZ
M,8,UX,,,UY,UZ,ROTX,ROTY,ROTZ
M,12,UX,,,UY,UZ,ROTX,ROTY,ROTZ

SAVE  ! save .db file
SOLVE ! generate the matrices
FINISH
```

2.5 Advanced usage

2.5.1 DOE & general organization in steps

The SDT `fe_range` architecture supports the generic definition of numerical experiments.

- 10 Import linear model (SDT/OpenFEM format)
- 20 Define non-linear properties `NLdata` for a group of elements (non-linear springs CBUSH, contact surfaces, zero thickness element, StressCut)
- 30 Define computational range and options

- 40 Solve and save results
- 50 Post-process automatically

```
model =  % SDT/OpenFEM format
     Node: [2x7 double]
      Elt: [6x9 double]
    Stack: {2x3 cell}
model.Stack = % List of properties
   % Case : Store boundary conditions and load
    'case'    'Case 1'        [1x1 struct]
   % Pro : properties of a group of non-linear elements
    'pro'     'nl_pro2001'    [1x1 struct]
NL=stack_get(model,'','nl_pro2001','get')
      type: 'p_spring'
        il: [2001 2.2503e-02] % Usual elastic properties
    NLdata: [1x1 struct]

NL.NLdata=  % Input format for non linearity data (CLIMA-HBM)
        type: 'nl_inout'
          Fu: '@(x)-.01*x.^3'
        Sens: 'cubicSpring'
     keepLin: 0
```

2.6 External links

References to external documents. In SDT use `sdtweb('ref')` to open the page.

- `ParamEdit` standard SDT parameter extraction `cingui`
- `fe_time` time integration `fe_time`.
- `staticNewton` non-linear static computation `fe_time`.
- `newmark` linear Newmark solver, `NLNewmark` nonlinear Newmark solver`fe_time`.
- `m_elastic` material property function , `m_elastic`
- `fe_caseg` assembly and GUI complement `fe_caseg`
- `ConnectionScrew`, `Assemble` assembly and GUI complement `fe_caseg`
- end

3

Function reference

Contents

List of the SDT-HBM module functions	
Function	Description
d_hbm	Open source HBM examples
d_tdoe	Open source time DOE examples
hbm_solve	Solving tools
hbm_post	Post process tools
hbmui	User interface
hbm_utils	Maintenance

d_hbm

Purpose

Open source examples for HBM

Syntax

```
d_hbm
```

`TestBeamNL`

Simple bending beam with localized non-linearity.

```
model=d_hbm('testBoltedJoint')
```

d_tdoe

Purpose

Open source examples for time experiments.

Syntax

```
d_tdoe % opens tag list of tutorials
```

`MeshCfg URN definition of meshes`

The handling of mesh configuration is done by `d_mesh MeshCfg`. The convention is that the mesh URN is of the form `Mesh:Case:NL`. Thus `d_hbm(0D):DofSet:0Dm1t` says that

- the `d_hbm('Mesh0D')` command is called to obtain the mesh
- the `d_hbm('CaseDofSet')` command is called to set case information (loads, sensors, ...)
- the `d_hbm('NL0Dm1t')` command is called to set non-linearity information. Here `0Dm1t` refers to a specific material.

`SimuCfg URN definition of time experiment`

The handling of time simulation configuration is done by `d_fetime SimuCfg`. Different conventions are being developed for different types of experiments.
`SteppedSine{.5,1,10}:C0{0,15}:C1{2.5,10}` is used to characterize stepped sine tests. The URN is split as follows

- `SteppedSine{.5,1,10}` gives the target frequencies in Hz.
- `C0{0,15}` gives the harmonic 0 or static offset. xxx scaling convention
- `C1{2.5,10}` gives the first harmonic or time signal of the form $cos(\omega t)$
- xxx 'NperPer',2e3,'Nper',1,'iteStab',20

hbmui

Purpose

Graphical user interface for the HBM solver (requires SDT)

Syntax

```
hbmui
```

Description

`hbmui` operates the GUI for HBM simulation procedures including pre, post treatment and simulation runs.

Commands

`Hide`

Not do display the GUI while running the HBM module.
`hbmui('hide');`
By default the GUI gets opened when the module is first loaded. Use this as the first call to the module to prevent the GUI from appearing. The GUI will remain hidden until an explicit call to `hbmui` is performed.

`Init[,Project,Post,...]`

Initializes the GUI and specific tabs. If the tab already exists, the display will be switched to the existing data.

```
hbmui init
hbmui initProject
```

The following command options are supported

- `-Reset` To reset the full GUI before the asked initialization.
- `-resetCurTab` To reset the tab specified for initialization.
- `-noTab` To initialize tab data without actually displaying it.

`PARAM[.`*Tab*`,UI]`

Access to GUI parameter values.
`RO=hbmui('PARAM.'`*Tab*`')'`
The output is a `struct RO` with fields corresponding to the parameter names in the tab *Tab*, and values interpreted from the current GUI state. By default an error will be issued if the mentioned tab has not been initialized. The following command options are supported

- `-safe` Not to generate an error if the tab has not been initialized, but rather return the default values.

- `-r1j` To recover the parameter underlying java object.

- `.Par` To recover parameter named *Par* only.

Command `PARAMUI` returns the complete application UI structure.

`Set[Project,Post,...]`

Script version of GUI parameters sets.
`hbmui('setTab', struct('Par' ,'Val,...));`.
Tab is the tab name containing the parameter to set, *Par* is the parameter name, *Val* is the value assigned to the parameter. To recover parameters names, see `hbmui PARAM`.
To trigger an action linked to a push button, the value `do` must be assigned, *e.g.*
`hbmui('setPost',struct('refresh','do'));`

Tabs

Project

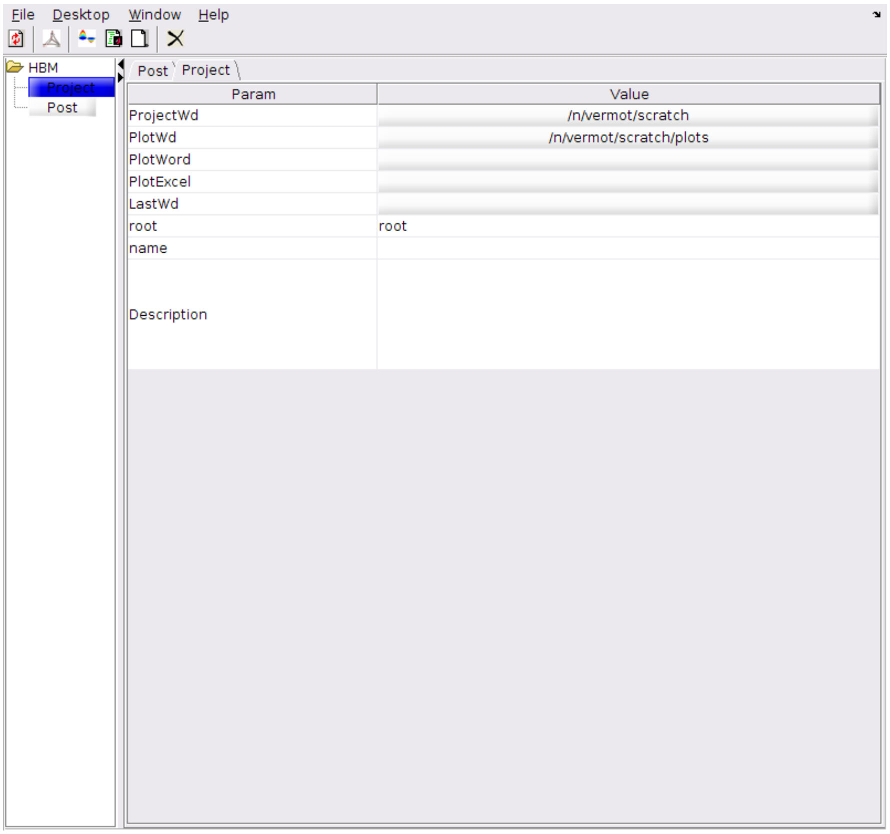


Figure 3.1: `Project` tab in initial state

Post

The `Post` tab handles post-treatment procedures allowing data export and display. In display mode it is linked to an HBM result object whose state can be altered through the GUI. To be used results must have been stored in the application.
The initial `Post` tab is presented figure 3.2,

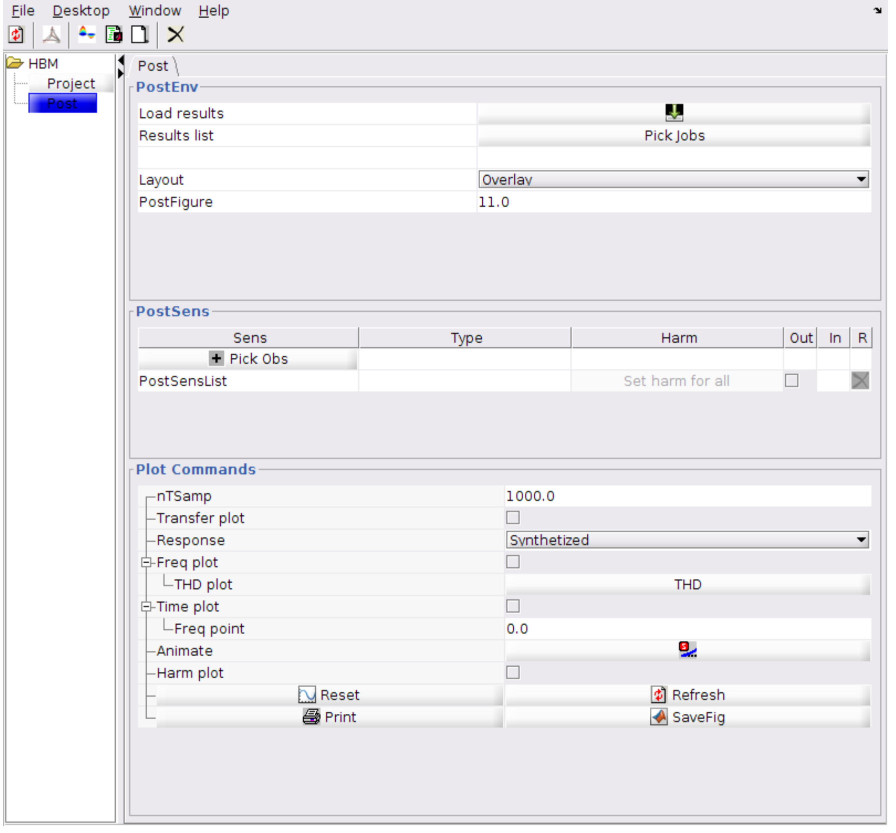


Figure 3.2: `Post` tab in initial state

It features three sections

- `PostEnv` Handles the post-treatment environment, namely the selected job, with the possibility to load one, the `iiplot` figure number to host display.
- `PostSens` Handles the observation stack, `PostSensList`. The first two lines allow handling the following ones that correspond to the actual observations. The first line proposes an interactive definition of a new post-treatment, see `hbmui PostDlgSensPick`. The second one is header to the list, with the possibility to apply changes to all following list entries regarding harmonic selection `Set harm for all`, selection toggle or full observation list deletion. The table features 5 columns
  - `Sens` Provides the observation set label.
  - `Type` Provides the type of signal, either `disp`, `vel` or `acc` for respectively a displacement, velocity, or acceleration signal.

- `Harm` Provides the harmonics retained for the post-treatment. The input will be treated as a subset of the job output harmonics. It can set either as a comma separated integer list, or a token (`all`, `even`, `odd` being supported).
- `Out` To toggle enabling of the current observation. If unchecked the observation list will be ignored in the post-treatments.
- `In` To select the current line as an entry for transfer displays. A maximum of one observation line can be selected as an input.
- `R` To suppress the corresponding observation line.

- `Post Commands` Drives the display options and commands. The following parameters can be set (presented in the format *`Tab.Par`*)

  - `Post.RespSyn` The type of synthesis to be performed, either `Synthesized` to synthesize the response on observation with all retained harmonics, `byHarm` to generate an observation response per harmonic, or `byShape` to generate an observation response per time shape.
  - `Post.nTSamp` Provides the number of time samples for transient displays.
  - `Post.FreqPlot` To select a maximum amplitude response as function of the frequency plot.
  - `Post.TimePlot` To select a transient response plot at a given frequency.
  - `Post.FreqPoint` To provide a frequency point associated to the transient response plot.
  - `Post.HarmPlot` To select a harmonic contribution bar graph at a given frequency.
  - `Post.TransferPlot` To generate a transfer response between two observations.

  The following commands can be launched

  - `Post.PlotReset` To reset the `iiplot` figure with the current setup.
  - `Post.PlotRefresh` To refresh the display with the current setup.
  - `Post.PlotPrint` To print the the display into a plot, support for automatic reporting is provided through the `Porject` tab setup.
  - `Post.PlotSaveFig` To save a figure containing the current display.
  - `Post.TimeAnim` To animate the transient response associated to the current setup.

PostDlgSensPick

The SensPick dialog box provides tools for an interactive definition of observations based on jobs stored in the application. When clicking on the PickObs button in the PostSens section of the hbmui Post tab, the dialog box presented in figure 3.3 opens.

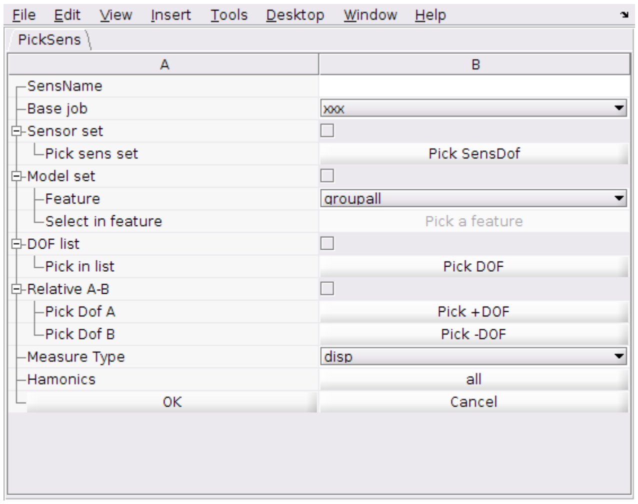


Figure 3.3: Observation selection/generation dialog

One then gets the possibility to define an observation with the following options

- SensName A string defining the observation name that will be used as label in displays.
- BaseJob The job result with which the observation will be generated, used as context in the dialog following options.
- The type of observation, to be checked in the list presented below. The choice is exclusive and will expand the adequate options.
  - Sensor set To use a SensDof entry (see sdtweb sensors) that is present in the job model. Use the PickSensDof button to access the list of available entries and pick one.
  - Model set To use an integrated model based element selection. The automated selection is based on usual model accessible information. On can use a feature (as type of information) between EltSet, Mat, Pro, or groupall. Use the Pick a feature button to access respectively the EltId set list, the MatId list or the ProId list declared on the model. The groupall feature directly selects all elements in the model and thus do not need further selection.

  - `DOF list` To use a model DOF. The `Pick DOF` button then provides the complete list of available DOF for the user to pick one.
  - `Relative A-B` To generate an observation based on a relative movement between two model DOFs, $y = q_1 - q_2$. Use the `Pick +DOF` to access the list of available DOF and select q_1. Use the `Pick -DOF` to access the list of available DOF and select q_2.

- `Measure Type` To provide the type of signal to generate, either `disp` for displacement, `vel` for velocity, or `acc` for acceleration.

- `Harmonics` To provide a sub-selection of harmonics retained for the post-treatment synthesis. Click on the button defaulted to `all` to access the list of available choices. Besides the list of harmonics present in the job result, one can choose `all` to retain all harmonics, `odd` to retain all odd harmonics only, or `even` to retain all even harmonics only.

- `OK` To validate the input and proceed to the observation generation.

- `Cancel` To cancel the current input and close the dialog.

hbm_post

Purpose

Commands for result post-processing. This functions provides post-treatment commands and handles the HBM result object.

Description

ZTraj[,Get,GetBnl,Set,SetDef]

Trajectory generation as harmonic result structures Zcurve from synthesis of given shapes.

- Command ZTraj generates a harmonic response associated to a given trajectory.
  `Z=hbm_post('ZTraj',model,def);`

  model is a standard SDT-NL model, def is a def curve expressed on the model DOF, typically a modeshape. The output is a HBM output Z. The model is HBM-assembled and data are initialized based on model.Stack{info,HBMOpt} or using the default options output by hbm_solve Opt. If the def curve is real, the output will be initialized with the shape on the c1 harmonic, the pulsation is initialized by def.data(:,1). If complex, the c1 harmonic is set to the real part and s1 to the opposite of the imaginary part, the pulsation is initialized by def.data(:,1).

  The output of hbm_post ZTraj cannot directly be exploited by other ZTraj commands as an observation (see hbm_solve AddPost) must be declared prior to generating the the HBM results object allowing response synthesis. The following command options are supported

  - -res to output the result in .Res format.
  - -reAss to force reassembly of the model.
  - -TKT to perform model projection on def.TR.
  - -harm to specify harmonics to be used for the results format (by default the first harmonic is used).
  - -useq0 in conjunction with -TKT to use the static state model.Stack{curve,q0} in the trajectory, the static state will increase def.TR prior to projection.
  - -q0inTR to detect the zero harmonic in def.TR (as with def.data(:,1)<1), thus initializing different generalized harmonic DOF for the zero harmonic and the others. This option is handled internally if useq0 is used.

- Command ZTrajSet updates the HBM results object Z with different harmonics.
  `XF=hbm_post('ZTrajSet',XF,RA);`
  XF is the HBM results object, RA is a structure with field .harm defining the new set of harmonics.

- Command `ZTrajSetDef` only updates the HBM curve `Z` with a new trajectory based on the same harmonic DOF than the initial one.
  `XF=hbm_post('ZTrajSetDef',XF,struct,opt,struct,d1);`, with `XF` the HBM results object, `opt` the opt structure, `d1` the new shape.

- Command `ZTrajGet` synthesizes and outputs a transient NL trajectory (in the standard `fe_time`NL format) based on one of the state of the provided HBM results object.
  `def=hbm_post('ZTrajGet',XF,RO);`. The following command options are supported

  - `nTSamp`*`val`* to set the number of time samples to *`val`* in the output.
  - `NoT` to generate a trajectory based on the model DOF.
  - `iMode`*`ind`* to select the state index *`ind`* in the HBM results object.
  - `coef`*`val`* to apply an amplitude coefficient *`val`* to the state.
  - `NeelUNL`*`val`* to output the non-linearity observations. Set *`val`* to a two digit number `[unl vnl]` set to `1` if needed, `0` otherwise to obtain `def.FNL.unl` and/or `def.FNL.vnl`.
  - `initnl` to perform and keep initializations necessary to optimized trajectory update by linear state coefficient application with `hbm_solve @defUpCoef`.
  - `NoStat` to ignore `unl0` and `vnl0` in the observation and evaluation of non-linear forces.

- Command `ZTrajGetBnl` outputs the non-linear harmonic forces based on the provided HBM results object.
  `bnl=hbm_post('ZTrajGetBnl',XF,RO);`.
  `XF` is a HBM results object, `RO` is an option structure with optional field `.nTSamp` to specify the time sampling number used to evaluate the harmonic non-linear forces.

`AddPost`

Handles observations declarations for post-treatments from the HBM curve output to the HBM results object.

`hbm_post('AddPost',job,data);` is used to handle outputs stored in the application. `job` is the name on which the observation will be applied (entry in PA.Stack). `data` provides the observation, it can be of the following types

- A DOF list in `numeric format`

- An observation `struct`, with fields `.cta` an observation matrix and `.DOF` the associated DOF vector. `struct('cta',1,'DOF',21.03,'name','tip')`. Appends the SensDof entry in the model `PA.Stack{job}`.

- A model. If field .DOF is present, it is directly used, otherwise DOF are obtained from the .Node field.

For all cases, a Sens data structure is generated and a SensDof entry is added to the model PA.Stack{job}2. The GUI then adds the observation to the available observation list.
Manual handling outside the GUI requires the use of a results structure RE as a third argument. The same operations are performed, the observation list stored in RE.PostSensList being updated RE=hbm_post('AddPost',lab,data,RE);.

Init

Initializes the HBM result object.
XF=hbm_post('Init',UI,ci);.
UI is a variable pointing to the HBM solver output data. It can be left empty, in such case it is initialized by UI=hbmui('PARAMUI'), or it is a results structure with mandatory fields .Res (see hbm_post ZTraj-Res) and .PostSensList (see hbm_postAddPost). ci is a iiplot object that will host the results display. It can be left empty not to display results.
If results are stored in the application, the HBM result object associated to the current application state can be recovered using XF=hbm_post('init').
Manual initialization with no display can be performed from an assembled harmonic model mo1 and a HBM curve output structure Z using

```
RE=struct('Res',{{Z.name,{Z,mo1}}});
RE=hbm_post('addPost',Z.name,mo1.DOF,RE);
XF=hbm_post('init',RE,[]);
```

HBM results object

The HBM results object provides methods to synthesize transfers or transient trajectories based on a HBM result, it is based on the curvemodel object that is a SDT curve wrapper. The result object is initialized using hbm_post Init.

XF.[GetData,X,Y,Xlab]

Provides access to the resolved content of the HBM result object XF in its current state.
X=XF.X;
Warning: depending on the current object state, recovering the full output can be a very intensive task and may generate a very large volume of data !
The .GetData method returns the full curve structure synthesized for the current object state. The other methods only return the field corresponding to their name.

`XF.set`

HBM results object options handling.
`XF.set('nTSamp',100);`
For an app linked object, the options are available in the GUI `Post` tab. This programmatic way can be used in scripts interacting with the GUI, or with results objects not linked with the GUI.

`XF.Stack`

Provides access to HBM results context data. The object mode can be programmatically altered through this way.
`XF.Stack'type'=val`;
Accepted types for `val` are

- `Freq` To handle maximum amplitude frequency responses.
- `Time` To handle transient responses.
- `Harm` To handle Harmonic contribution responses.
- `FFT-1` To handle FFT of the first harmonic transient response.

hbm_solve

Purpose

HBM Solver and base utilities commands.

Description

Variables used during the resolution are the opt structure

`AFMap`

Performs amplitude/frequency response MAPS, possibly in a given subspace.

`Z=hbm_solve('AFMap',model,RO);`

`model` is a standard SDT-HBM model. By default the study is performed on the first harmonic of the model active DOF. It is possible to provide a customized `harmhdof` vector or even a reduced subspace by using a third argument `def` as a structure with fields

- `.TR` a structure providing a physical reduction basis, with mandatory fields `.TR.DOF` the full DOF vector and `.TR.def` the Rayleigh-Ritz vectors stored in column.
- `.hdof` a `harmhdof` vector based on the generalized DOF defined by `.TR`.

`RO` is a structure with fields (that can also be provided in a preemptive way in the `RO` structure)

- `.Freq` frequency vector in Hz.
- `.Amp` amplitude (scaling coefficient applied to the loads). Default equal to 1.
- `-bnl` to store and output the NL force vector
- `-iter` to perform resolution iterations. By default the response is based on the response prediction induced by the non-linear forces generated by the linear response trajectory.
- `-z0` to store and output the linear response.
- `-itInit`*`stra`* to alter the initialization strategy, set *`stra`* to either `0` to initialize at a given amplitude by the scaled linear initial state a the first amplitude, or `1` to keep the current response of the previous state.
- `-fscan` to perform a direct frequency scan for a given amplitude and interpolate the result on the `.Freq` input.
- `RO.pList` field can be provided, defines the parameters order in a cell array, to be used in the multiple loop from external to internal. By default set to `{Amp, Freq}`.
- `RO.harm` defines the harmonic vector to be used, by default set to `1`.

- `model.Stack{'info','HBMOpt'}` Provides a custom opt structure. Beware that many fields are redefined to perform this resolution.

Output `Z` is a standard SDT-HBM `Zcurve` with multiple dimensions corresponding to `Hdof`, `Freq` and `Amp`.

`Assemble[call,init,exit]`

Provides and performs SDT-HBM specific pre-post assembly operations. The assembly call to be passed to `fe_case` is provided by

```
st=hbm_solve('AssembleCall');
```

`AssembleInit` and `AssembleExit` are internal calls handling non-linearity initialization and proper load definitions.

`fe_time[,cleanup]`

Command `fe_time` is called back by `fe_time`as the base HBM solver, defined by field `opt.Method` in the opt structure.

Command `fe_timeCleanup` performs base post-treatments to the raw solver output and handles direct storage in interaction with `hbmui`. By default the output will be stored in the application, and base results can directly be displayed in `iiplot`.
This command is usually performed as an internal command at solver exit based on the field `opt.FinalCleanupFcn` in the opt structure. The command then expects that the caller uses variables `out`, `opt` and `model` to respectively store the solver output as a HBM curve output, the solver running options (opt) and the assembled model used by the solver. An external call is also possible, using `out=hbm_solve('fe_timeCleanup',out,opt,model)`.
The following command options are supported

- `reset` to reset `iiplot` before display.
- `NoPlot` not to display the results in `iiplot`.
- `NoUI` not to store the results in the application.
- `ExitFcn`*cbk* to perform additional custom operations at the end of the cleanup procedure, the string *cbk* will directly be called by the `eval` function.
- `-rethrow` to output the result data structure.
- `-cf`*val* to provide a specific `iiplot` figure. If *val* is not strictly positive the default `iiplot` figure (or last active one) will be used, otherwise the figure with given positive handle will be used.

harm[lab,hdof,place,c]

Harmonic handling utilities. This series of functions provide tools for HDOF definition handling and matching.

- harmHdof defines a harmonic DOF (HDOF) vector.
  hdof=hbm_solve('harmHdof',model,harm);. model is a standard SDT model from which the active DOF will be recovered, using model.DOF if present, or fe_casegetTDof. It is possible to provide a DOF vector instead of a model. harm provides the harmonics numbers to be used, set to 1 if omitted. To select only specific time functions, it is possible to use a regular expression token as a third argument.

- harmC performs HDOF localization utilities.
  c=hbm_solve('harmC',hdof,sdof) provides an observation matrix c to observe harmonic DOf subset sdof in hdof.
  sdof=hbm_solve('harmC',hdof,sdof,typ,in) respectively provides

  - typ='dof', in=1 the intersection of hdof and sdof .
  - typ='ind', in=1 the indices in hdof intersecting with sdof.
  - typ='dof', in=2 the harmonic DOF of hdof not present in sdof.
  - typ='ind', in=2 the indices in hdof no present in sdof.

  Command option hID performs the match on the time function label only.

- harmPlace Places a harmonic def (defined on a set of harmonic DOF) on the set of harmonic DOF hdof.
  d1=hbm_solve('harmPlace',hdof,def);.

- harmLab A rather internal subfunction generating time dimension labels.
  lab=hbm_solve('harmLab',1:3).

```
model=demosdt('demoUBeam');
hdof=hbm_solve('harmHdof',model,0:3); % adof, {c0,c1,s1,c2,s2,c3,s3}
hdof1=hbm_solve('harmHdof',model,1,'s'); % adof, s1

i1=hbm_solve('harmC',hdof,hdof1,'ind',1);
sdof=hbm_solve('harmChID',hdof,'c1','dof',1);
isequal(hdof1,sdof)
```

Opt

Solver options handling. Reference to data fields is provided in opt for initialization and opt for values used during the solve.

```
opt=hbm_solve('opt');
opt=hbm_solve('opt fstart=.1 fend=100',opt);
RA=struct('fstart',.5,'fend',100,'dsmin',.001);
opt=hbm_solve('opt',[],RA);
opt=hbm_solve('opt',opt,RA);
```

Reduce

Performs model reduction with proper handling of non-linearities.
`[model,Case,Load]=hbm_solve('Reduce',model,TR);`
`model` is a SDT-HBM model, `TR` is a `def` structure defining the reduction basis.
The model will be fully assembled then reduced, it is possible to provide a pre-assembled model. To do so, it is necessary to define the associated case by providing it in a third argument. The output is an assembled reduced model complying with initialization of SDT-HBM resolution procedures.

@abscHBM

Performs a state prediction based on an interpolated curvilinear frequency step.
`[Z,opt]=abscHBM(Z0,opt,j1);`
`Z0` is the initial state provided to the solver, `opt` is the internal solver structure, `j1` is an interaction indicator.
The outputs are `Z` a predicted state, and `opt` with updated curvilinear frequency step `opt.dS`.
For the first iteration `j1==1`, the initial state `Z0` is output.
For the fixed strategy `opt.dsOpt.step==0`, the curvilinear frequency step is set to `0.5*opt.dsOpt.dsfix`, and the predicted state is the initial state `Z0` with the new frequency.
For the Lagrange interpolation strategy `opt.dsOpt.step==1`, the curvilinear frequency step is defined as being between `opt.dsOpt.dsmin` and `opt.dsOpt.dsmax` and updated to optimize the number of iterations towards the target `opt.dsOpt.iopt`. The update is performed by applying a multiplicative coefficient to the current frequency step `opt.dS` as the ratio between the optimal iteration number `opt.dsOpt.iopt` and the last number of iterations `opt.ite`. The multiplicative coefficient is bounded between `opt.dsOpt.bmin` and `opt.dsOpt.bmax`. The predicted state is the result of the Lagrange interpolation of degree `opt.dsOpt.Ldeg` of the last states at the incremented frequency.

@ATimesZ

Computes the linear dynamic harmonic forces `o1=A*z0` in an implicit manner (*i.e.* without actually building the dynamic harmonic stiffness matrix).
`o1=ATimesZ(model,Case,opt,z0)`
The output `o1` is the linear dynamic harmonic forces.

@buildA

Initialization and/or generation of the harmonic dynamic stiffness.
`opt=buildA(model,Case,opt,Z,i1)`.
`model` is a SDT-HBM assembled model, `Case` is the associated case, `opt` is the internal solver running option structure, `Z` is the current harmonic state; these variables must have been initialized by `hbm_solve InitHBM`. `i1` is the output option:

- `i1=0` will output `opt` with field `opt.A` initialized as the dynamic harmonic stiffness matrix.
- `i1=1` will output `opt=A` as the dynamic harmonic stiffness matrix.
- `i1=2` performs optimized initialization of the dynamic stiffness matrix as a cell array of component matrices, so that the frequency dependency can be represented as a weighted sum of the component matrices. If `opt.A` is empty an initialization at unit frequency is performed, if `opt.A` is a cell array the actual dynamic stiffness matrix is directly output (`opt=A`).

The frequency is recovered from context, either `opt.Opt.fvar==1` and the pulsation is taken as the last value of the harmonic state (`w=Z(end)`), or `opt.Opt.fvar==0` and the pulsation is directly taken as `w=opt.w`.
The output `opt` is either the running option structure with filled `opt.A` or directly the harmonic stiffness matrix.

@buildCHarm

Generation of interlaced harmonic observation or command matrices.
`[c,harm]=buildCHarm(c,opt,typ)`.
`c` is an observation or command matrix, `opt` is the internal solver running options that must have been initialized by `hbm_solve InitHBM`, `typ` provides the type of matrix `c`. Either `typ='c'` to declare an observation matrix, or `typ='b'` to declare a command matrix.
The output is the interlaced observation or command matrix `c`. Line ordering is interlaced (*i.e.*) the sequence is first the harmonics, then the initial line order. `harm` is the retained harmonics. For command matrices, Fourier coefficients are applied to the matrix terms.
It is possible to define a field `opt.subH` with a sub-set of harmonics to be used for the observation generation, independently from the initial `opt.harm` field that is used by default.

`@buildHkt`

Generation of time function space.
`opt=buildHkt(opt);`
`opt` is the internal solver running option initialized by `hbm_solve InitHBM`.
The output is the internal solver running option `opt` with updated fields `opt.Hkt`, `opt.dHkt` and `opt.Htk = opt.Hkt'`, that are the time harmonic components of (1.61). `opt.N` and `opt.harm` (representing k/ν) are mandatory fields.

`@buildLoad`

Prepares the external load structure for the HBM solver, based on the usual transient Load representation of `fe_time`.
`Load=buildLoad(Load);`
The output has a resolved field `.adof` providing the harmonic DOF used to described the external load, a command matrix `.b`, a field `.curve` providing the optional variations of external loads with the frequency.
`Load` is a load structure complying to time simulations. See `fe_load`, `fe_load buildu`.

`@ctaSubH`

Integrated generation of a subharmonic observation matrix based on a HBM output curve.
`opt=ctaSubH(cta,d0,subH);`
`cta` is an observation matrix, `d0` is an HBM output data structure, `subH` is a sub-harmonic selection.
`subH` is either directly a vector of harmonics, or a string token set to

- `all` to retain all harmonics of `d0.harm`.
- `odd` to retain only the odd harmonics of `d0.harm`.
- `even` to retain only the even harmonics of `d0.harm`.

The output is the `opt` data structure with added or updated fields `.cta`, `opt.iadof`, `opt.idof`, `opt.harm`, `opt.hVect`, `opt.hId` to comply with the subharmonic selection.

`@defUpCoef`

Optimized trajectory linear amplitude change.
`d2=defUpCoef(d2,Amp);`
`d2` is a trajectory initialized the `hbm_post ZTrajGet-initnl`, `Amp` is a scalar amplitude coefficient.
The output is `d2` the trajectory linearly set to amplitude `Amp`, relative to `d2.coef`, the initial amplitude.
The trajectory and non-linearity observations are updated using `Amp/d2.coef`, non-linear forces are re-evaluated to provide a linearly updated trajectory.

InitHBM

The subfunction @intHBM performs HBM solver specific initialization based on a SDT model with non-linearities. [model,Case,opt,Z0,Zf]=initHBM(model,Case,opt,u,fext);
model is a standard fully assembled SDT-NL model, Case is the corresponding resolved Case structure, opt is the solver running option structure opt, u is a system state expressed on Case.DOF, usually a static state, fext is the external Load structure expressed on Case.DOF.
model, Case and fext should be compliant to the SDT-HBM data structures, and can be typically obtained from usual models with the provided hbm_solve AssembleCall: [model,Case,fext]=fe_case(hbm
The outputs are data ready to be used in HBM solvers, opt is completed, Z0 is the initial state, Zf is the current harmonic load vector.
The initialization procedure

- prepares the internal solver running parameters,
- prepares system matrices and initializes the harmonic dynamic stiffness structures,
- initializes the initial harmonic state and state buffers for prediction and interpolation,
- builds external forces harmonic vectors,
- builds the harmonic interlaced observation and command matrices of each non-linearity.

@iterHBM

Performs an iterative resolution for a given initial state.
[Z,ki,opt]=iterHBM(ki,Zf,Z,model,Case,opt,j1);
ki is for the moment reset internally and can be provided empty. Zf is the harmonic load vector, Z is the initial harmonic state, model is the assembled model, Case the corresponding case structure, opt the solver running options. All these variables must have been initialized with hbm_solveInitHBM. j1 is a step indicator, also used in the base solver to know whether the state buffer has been fully filled.
The outputs are Z the resolved HBM state, ki the current Jacobian, and opt the internal solver option structure.

@getZf

Generates the harmonic external forces vector from the Load context. This is used by hbm_solve @resHBM.
Zf=getZf(Zf,opt);.
Zf is the current load data structure, or a resolved harmonic vector. Nothing is performed in the latter case. opt is the internal solver running option. In particular the current pulsation is found in opt.w.

@outputInitHBM

Initializes the solver output structure, to be filled by `hbm_solve outputHBMFcn`.
`[out,opt]=outputInitHBM(model,Case,opt);`
`model` is unused at the moment, `Case` is the case corresponding to the harmonic model, `opt` is the internal solver running option. These variables must have been initialized by `hbm_solve InitHBM`.

@outputHBMFcn

Output data structure (pointer addressed) filling.
`outputHBMFcn(out,Z,opt);`
`out` is an output data structure initialized by `hbm_solve @outputInitHBM`, `Z` is the current state to be possibly stored, `opt` is the internal running option solver.
There is not output associated to this command as input structure `out` fields are handled by pointer. The current state is stored if its associated frequency has changed in absolute value by more than `opt.SaveFreq.fStep` from the last saved frequency point (`out.X2(out.cur(2))`). The frequency is recovered from context, either `opt.Opt.fvar==1` and the pulsation is taken as the last value of the harmonic state (`w=Z(end)`), or `opt.Opt.fvar==0` and the pulsation is directly taken as `w=opt.j1`. In this case, `opt.j1` is differentiated from `opt.w` to allow customized handling of the saving strategy.

@resHBM

Computation of the harmonic balance residue and associated finite differences Jacobian.
`[r,ki]=resHBM(Zf,Z,model,Case,opt,ki,jite);`
`Zf` is the harmonic load vector, `Z` is the current harmonic state, `model` is the harmonic model, `Case` is the corresponding case, `opt` is the internal solver running option, `ki` is the current Jacobian, `jite` is a current iteration indicator.
The outputs are `r` the harmonic residue, and `ki` the associated Jacobian.
The Jacobian is computed by finite differences using a dZ amplitude defined by `opt.epsi` and initialized at `1e-9`. It is computed if `ki` is empty, or if `opt.JacobianUpdate` is not null.
It is possible to obtaind the external harmonic forces of the current model by setting `Z` to empty. In such case, the non-linear forces will be obtained from `model.FNL` and summed with `-Zf` in the output `r`.

hbm_utils

Purpose

Utilities used for HBM development and administration.

Syntax

```
hbm_utils CommandString
hbm_utils('CommandString')
```

Description

hbm_utils deals with the paths handling (Path command), the generation of the documentation (Latex command) and other utilities.

Path

hbm_utils('Path') fixes MATLAB path and sdtweb('_path') to include DYNAVOIE based on the result of which('hbm_utils'). The expected directory structure is detailed in section 2.1 .
Note that you should

Help

You can automatically update, your documentation files using hbm_utils('HelpGet') which will get a zip file from the server and decompress it into DynRootDir/help. The help contains both the PDF and HTML files which can be opened with the sdtweb function, for example:

```
sdtweb('hbm_utils') % requires sdtweb >1.50
```

Note that you possibly have to configure your proxy address in your Matlab preferences (File/Preferences/

Latex,Hevea

The following commands are used by SDTools for the maintenance

- Latex compiles the documentation using pdflatex.
- pdf opens the manual in a browser window.
- hbm_util('HelpPut') updates the zip file containing the documentation

Verbose_Mode

Verbose mode mecanism. Now one can different levels of warning:

```
hbm_utils('Verbose_Mode',-1) % SILENT
hbm_utils('Verbose_Mode',0)  % NORMAL
hbm_utils('Verbose_Mode',1)  % VERBOSE
```

Distrib

DistribGen is used by SDTools to generate a protected copy of DYNAVOIE, which can be sent as a zip file. DistribPatch is used by SDTools to generate a SDT patch relative to the last reference version of SDT.

nl_spring

Purpose

Non linear links/force modeling for time simulation

Syntax

```
model=nl_spring('tab',model);
...= nl_spring('command', ...)
```

Description

nl_spring supports non-linear connections and loads for transient analysis. Non linear springs between 2 DOF (see nlspring). loads which depend on DOF values (see DofKuva, DofV), springs between 2 nodes in different bases (see RotCenter), etc. ...). A full list of non-linearities is given in nllist

Standard non-linear simulations are handled by nl_solve. Below is a description of the inner mechanisms of a non-linear simulation with the non-linear toolbox.

After the non linearity definition, a proper TimeOpt is required to set the good fe_time calls to perform a non linear Newmark time integration. A default TimeOpt can be set using nl_spring TimeOpt. It is possible to save transient results on the fly using a properFinalCleanup call, see nl_spring fe_timeCleanupCall , and to reload the same results using fe_simul fe_timeLoad.

The following steps are required for a time simulation

- Definition of non-linear properties. These are stored as pro entries of the model stack. The associated property function must handle non-linearities which is currently only the case for p_spring and p_contact.

  A non-linearity is always associated with elements or superelements (typically a celas element. A given group of elements can only be associated with a single non-linearity type.

  The information needed to describe the non linearity is stored in a .NLdata field.

- Model initialization using the an fe_case('assemble') call in fe_time, is followed by the building of a model.NL stack that describes all non-linearities of the model in a format that is suitable for efficient time domain integration. This translation is performed by the nl_spring NL command.

- Jacobian computation, see nl_spring NLJacobianUpdate.

- Residual computations are performed through mkl_utils. The nominal residual call is r=-fc; mkl_utils('residual', r,model,u,v,a,opt,Case);.

Supported non linearities

See nllist for supported non linearities, and nl_fun to add your own non-linearities.

`ConnectionBuild`

One can define a set of non linear links between 2 parts of a model using a call of the form
`[model,idof]=nl_spring('ConnectionBuild',model,data);`
`idof` is a second optional out argument. It returns the list of DOF concerned by links (it can be useful in order to reduce super elements keeping idof as interfaces DOF for instance). `data` contains all the information needed to define links. It is a 3 column stack like cell array. First column contains the string `'connection'`, the second the name of the non linear link described in the third column that contains a data structure with following fields:

- `.Ci` define nodes to connect in first (`.C1`) and second component (`.C2`). It can be a vector of NodeId or a screw data structure (slave nodes of the model nodes via `RBE3` links, see see `sdtweb('fe_case#connectionscrew')`.
- `.link` defines how to link component 1 to component 2. It is a 1x2 cell array. First cell defines the type of link (`'EqualDof'` or `'Celas'`) and the second gives information about the link. For celas link it is a standard element matrix row with 0 replacing NodeId : `[0 0 DofId1 DofId2 ProId EltId Kv Mv Cv Bv]`.
- `.NLdata` (optional) defines non linearity associated to celas link. See the list in list of supported NL. If this field is not present or empty, only linear link is considered.
- `.PID` (optional) is a 1x2 line vector that defines PID (second column of .Node matrix, see `sdtweb('node')` of connected node (1rst column for 1rst component).
- `.DID` (optional) is the same as above, defining DID (third column of .Node matrix, see `sdtweb('node')` of connected nodes.

Following example defines a model with a cylinder and a hole in a block. The cylinder is linked to the block by 3 celas preserving the pivot link.

```
mo1=demosdt('demoConnection-vol'); % meshes models
mo1=fe_case(mo1,'fixdof','base','z==-1'); % clamps the cylinder base
r1=struct('Origin',[0.5 0.5 0.5],'axis',[0 0 1],...
          'radius',.1,'rtol',.01,'length',1,'Npt',-3,...
          'ProId',111,'planes',[]); % Cylinder-side
r1=nl_spring('ConnectionCyl',r1); % defines planes
r3=r1; r3.ProId=1; % Block-side
link={'connection','link1',struct('C1',r3,'C2',r1,...
         'link',{{'celas',[0 0 12345 12345 1000 0 1e9]}})}; % Defines connection
[model,idof]=nl_spring('ConnectionBuild',mo1,link); % builds connection
cf=feplot(model); % displays in feplot
fecom promodelviewon;  fecom('curtab Cases','link1_2');
```

```
def=fe_eig(model,[5 20 1e3]); % computes the first 20 modes
if length(find(def.data<1e-3))>1; sdtw('_err','connection failed'); end
cf.def=def; fecom ColorDataAll % displays modes
```

See also `t_nlspring('2beam')` example.

`ConnectionCyl`

Utility to fill the `.planes` field of a cylinder connection in the standard connection screw data structure format (see `fe_caseg ConnectionScrew`).
`dataOut=nl_spring('ConnectionCyl',dataIn);`

The `dataIn` uses fields:

- `.Origin` origin of the cylinder axis, `.axis` orientation of the cylinder
- `.rtol` radius tolerance for cylinder selection.
- `.length` length of the cylinder.
- `.Npt` number of planes (equally distributed on the whole length). If Npt<0, ends of the cylinder are included in the connection points.
- `.ProId` ProId of the elements containing nodes to connect.

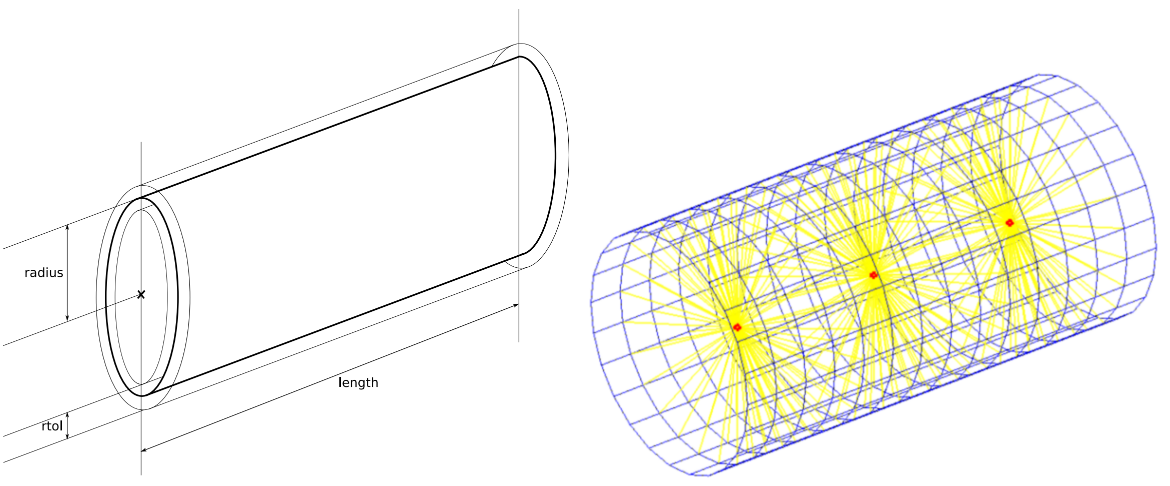


Figure 3.4: ConnectionCyl

InitV

`q0=nl_spring('InitV',model,d0,RO);`
`InitV` computes the initial static displacement and velocity associated to a DOF initial position and velocity. `d0` is a data structure with field `.DOF` containing the DofId where initial value is applied and `.def` containing initial displacement and velocity at this DOF. RO is a optional input argument data structure with following fields that define:

- `.dt` time step for time integration.
- `.dq` increment for initial vel computation.
- .Nv] number of time steps to reach d0.def(1) (displacement is imposed as a $0.5(1 - cos)$ time function on these time steps).
- `.Np` number of steps to stabilize at d0.def(1) and d0.def(1)+dq.

If input argument `RO` omitted, options are get from `'info' 'initvopt'` Stack entry. If there is no such entry, `InitV` parameters are computed using `-optim` process (see below).
Displacement at q0 and q0+dq is obtained meaning the last Np/10 steps of each stabilization period, and initial velocity is computed from those 2 displacements to match `d0.def(2)` at `d0.DOF`.

`[q0,RO]=nl_spring('InitV-optim',model,d0);` can be used to find input parameters `RO`. Optimization of dt and Np is performed from given or default values. Parameters dq and Nv are kept at given or default value. First dt is optimized. dt is increased (multiplied by 4) until time integration of the `InitV` process diverge and last dt that leads to convergence is kept. Then Np is increased by 100 steps until the deformation is converged on the stabilization periods, that is to say that a criteria taking in account standard deviation/mean of the deformation and the ratio of the last Np/10 steps upon previous Np/10 steps on each Np period is less than a tolerance (2.0).

See also `t_nlspring('2beam')` example.

NL

`model=nl_spring('NL',model)`
This command is used to build `.NL` field data for time integration from `NLdata` field in NL `p_spring` property entries in the input model Stack. The command option `-storefnl` can be used to specify the way of computing and storing a non linear effort associated to NL (for those which support it).

NLJacobianUpdate

`opt.Jacobian=nl_spring('JacobianCall')` returns the callback used to update or initialize the Jacobian `ki` used in iterative methods. This is the low level implementation of calls documented in nl_solve TgtMdl. The said Jacobian must take non-linearities into account and is thus of the form

$$k_j = [b]\left[\frac{\partial s_{nl}}{\partial u_{nl}}\right][c] \tag{3.1}$$

the output is controlled by the value of NL.Jacobian.

- 0 gives no Jacobian.
- 1 use finite differences to evaluate Jacobian.
- 2 fixed Jacobian.

For the case of a non-linear spring, the most important gradient of the tabulated law Fu is added as stiffness between the 2 DOF to the stiffness matrix and the most important gradient of Fv to the damping matrix.
For non-linear iterations in a Newmark scheme, the Jacobian is given by

```
ki=(model.K{3}+kj)+ (opt(2)/opt(1))/dt*(model.K{2}+cj) + 1/opt(1)/dt^2*model.K{1};
```

Accepted command options, associated to variants of the call are

- There are three outputs accessible, being [ki,mo1,C1]=nl_spring('NLJacobian'...).
- -noFact not to factorize the output Jacobian. This is useful if further actions are performed on the Jacobian after the standard call.
- -TangentMdl to return tangent model. It is assumed that model.K(1:3) correspond to M, C, and K (in this order). u and v variables of caller workspace can be needed.
- -TangentMdl-back to return a superelement containing the tangent matrices.
- -TangentMdl-back-sepKj to return a superelement containing the tangent matrices split by non linearities.
- -ener to compute for each def stored in model.d1 def structure (that is typically computed modes), some associated energies:
  - freq frequency in Hz.
  - damping damping ratio: $(\phi_j^T[C]\phi_j)/(2\omega_j)$.
  - enerK total strain energy: $\phi_j^T[K]\phi_j$.
  - enerC $\phi_j^T[K]\phi_j$.

- *NLlink*-enerK strain energy for each NL link: $\phi_j^T[K_{NLlink}]\phi_j$.
- *NLlink*-enerK for each NL link: $\phi_j^T[C_{NLlink}]\phi_j$.

SetPro

`model=nl_spring('SetPro ProId i ParamName1 Value1 ...',model)`
This command is used to change some nl_spring properties parameters. *i* is the ProId of corresponding p_spring property, *ParamName* the name of parameter to change (k for il(3), c for il(5) or the field name in NLdata) and *Value* the value to assign.
It is possible to define a new property by specifying an NLdata structure in third argument: `model=nl_spring('SetPro ProId i',model,NLdata)`. If the property already exists, the NLdata is interpreted as a string of parameters and parsed to define the fields specified in the given NLdata to the existing one. Command option Edit allows directly merging the existing NLdata to the provided NLdata with priority given to the new fields.

```
model=nl_spring('Demo1DOF');
% define a non linearity with partial definition of parameters and other by default
NLdata=nl_fun('db data 4') % standard NLdata defintion
% NLdata has fields data, Jacobian (by default) and type
% set in model
model=nl_spring('setpro proid201',model,NLdata);
% edit the nl_fun nl by string keyword
model=nl_spring('setpro proid201 data2',model);

% edit the nl_fun with struct input
% property will be parsed using nl_fun('paramedit')
model=nl_spring('setpro proid201',model,struct('Jacobian',2));
% field Jacobian has been edited, other fields are kept unchanged
model.Stack{end,3}.NLdata

model=nl_spring('setpro proid201',model,struct('NewField','test'));
% you can see that in this case NewField was not set
% as it is not referenced in the nl_fun parameters
model.Stack{end,3}.NLdata

% Force the with struct input with no check
model=nl_spring('setproedit proid201',model,struct('data',10,'NewField','test'));
% in this mode the NewField is propagated regardless of the
```

```
% standard nl_fun input
model.Stack{end,3}.NLdata
```

Standard NLdata structures depend on the non-linear function, see nllist for more details. They can be obtained through the nl_function command db, see nl_fun for more details.
In the case where

GetPro

```
pro=nl_spring('GetPro',model)
```

This command is used to get non linear properties in the model stack.

- Command option ID allows getting a specific non linear property by specifying its ProId.
- Command option type''nl_fun'' allows getting the non linear properties of a specific type. See nllist for more details on types of non-linearities.

Follow

The Follow mechanism can be used to observe some variable evolution during the time integration.
opt=fe_simul('Follow*i*',opt);

1st Follow consists in monitoring the number of iteration, the residual norm and displacement increment norm at each time step.

```
model=nl_spring('Demo1DOF')
opt=stack_get(model,'info','TimeOpt','GetData');
opt=fe_simul('Follow1',opt); % niter norm(r) norm(dq)
def=fe_time(opt,model);
```

2nd Follow consists in monitoring the def.FNL in iiplot. For the moment the mechanism is different (so note that you can't both tracker niter and FNL), and you only have to specify the field .FnlIiplot equal to 1 in the 'info','OutputOptions' stack entry of the input model, as in following example :

```
model=nl_spring('Demo1DOF');
r1=stack_get(model,'info','OutputOptions','GetData');
r1.FnlIiplot=1; % define FNL tracker
model=stack_set(model,'info','OutputOptions',r1);
opt=stack_get(model,'info','TimeOpt','GetData');
def=fe_time(opt,model);
```

TimeOpt

This command returns usual default TimeOpt for non-linear simulations. By default the output is the same as the TimeOptNLNewmark presented below. See also fe_time for TimeOpt definition details. Supported TimeOpt commands are

- TimeOptNLNewmark, or TimeOpt to obtain the TimeOpt for NLNewmark simulations. Use TimeOpt-gamm .51 to introduce numerical damping by directly giving gamma.
- TimeOptStat to perform static simulations (see also fe_time nl_solve).
- TimeOptTheta to perform time simulations with the θ-method (see fe_time). Numerical damping can be introduced using TimeOptTheta-alpha .05, the specified α value will be added to θ, so that the coefficient used in the simulations will be $\theta_1 = \theta + \alpha$.
- TimeOptExplicit to perform time simulations with the explicit Newmark scheme.

The following command options allows setting other TimeOpt fields to their desired value.

- dt*val* time step.
- ts*N* number of time steps.
- tend*val* optional end time
- tInit*val* initial time.
- AlphaR*val* a global Rayleigh damping mass coefficient (applied to the model total mass).
- BetaR*val* a global Rayleigh damping stiffness coefficient (applied to the model total stiffness).
- maxNout*N* requests an output subsampling strategy such that only *N* times equally spread over the simulation time span are output.
- RelTol*val* requests a specific relative tolerance for the convergence of iterative schemes.
- -gamma*val* requests a specific γ coefficient (default to .5) of the Newmark scheme. For the non explicit versions, β is adapted to ensure unconditional stability of the scheme.
- -theta*val* requests a specific θ coefficient (default to .5) of the Theta method.
- -acall*str* provides a series of command options applied to the AssembleCall generation.
- -fclean*str* provides a series of command options applied to the FinalCleanupFcn generation.
- -jcall*model* edits the jacobian call to allow late model modification.

Alternatively to providing all these command options in the command string, one can provide a MATLAB `struct` with equivalent fields as an additional argument.
By adding an SDT model as third argument, the generated `TimeOpt` will be directly integrated in the model, that will be output.
Sample calls :

```
% basic call
opt = nl_solve('TimeOpt dt1e-6 ts3e5 maxNout1e4 -acall"lumpedMass"');
% call with struct input
RO=struct('dt',1e-6,'ts',3e5,'maxNout',1e4,...
'acall','lumpedMass');
opt = nl_solve('TimeOptExplicit',RO);
% basic call with model input
model = nl_solve('TimeOpt dt1e-6 ts3e5 maxNout1e4 -acall"lumpedMass"',[],model);
% call with struct and model input
model = nl_solve('TimeOptExplicit',RO,model);
```

Convergence tests depend on the iteration algorithm and several behaviors can be obtained by modifying `RelTol`. In any case the absolute value of `RelTol` is used for the convergence test application; its sign is used to determine the convergence test to be used as described in the following.

- For algorithms using `iterNewton` as `IterFcn`, as is the case for methods `newmark` (explicit or not), `NLNewmark`, and `staticNewton`.
  - using `RelTol > 0` tests the convergence of the mechanical residue, relative to value `opt.nf`. If `opt.nf` is not provided, the scheme takes in input the norm of the external forces `fc` at the first time step, or if zero the norm of the first residue of the first time step. If still zero, `opt.nf` is set to `1`. This convergence test is the most widespread as it ensures mechanical stabilization. It is strongly recommended for static computations, or when using large time steps.
  - using `RelTol < 0` tests the convergence of the displacement correction, relative to the current displacement norm. The idea of this mode is to stop iterating if the correction becomes negligible, this is very useful to limit iterations with little impact on the results in transient simulations with small enough time steps. This must be used with care as this criterion does not imply that the mechanical residue is converged at the end of the time step, it is thus strongly advised to check results convergence.

  `iterNewton` does not support the use of `opt.cvg` yet.
- For algorithms using `itertheta_nl` as `IterFcn`, as is the case for method `theta`,

– using RelTol > 0 tests the convergence of the velocity field, its correction relative to the previous iteration velocity norm.

– using RelTol < 0 tests the convergence of the velocity field, and the model.FNL vector, their correction relative to the previous iteration norm. Stabilization of the model.FNL field may be difficult to attain and very sensitive as this vector can contain heterogeneous data, this mode is then not recommended by default, and use of opt.cvg should be preferred.

iterthetal_nl supports the use of opt.cvg, that forces iteration if set to 1. It is reset to zero at the start of each iteration, but any non-linearity can alter its value by using sp_util('setinput',opt.cvg,ones(1),zeros(1));. Each non-linearity can thus internally test the convergence of its fields of interest and apply a convergence veto if its convergence is not satisfied.

From standard fe_time simulations, the following TimeOpt fields are added or modified

- Jacobian field is modified to take into account non linearities, see NLJacobianUpdate.
- Residual field is modified to take into account non linearities, and to use mkl_utils to improve computation times, see sdtweb mkl_utils. This should be initialized by nl_spring('ResidualCall'
- AssembleCall field is modified, to perform non-linearities initialization after assembly. AssembleCal is the string passed to fe_case, generated by nl_spring('AssembleCall').
- OutputInit field is modified to also check non linearities and initialize non-linearities related outputs, this is a callback generated by nl_spring('OutputInitCall').
- FinalCleanUpFcn field is modified to perform cleanup on non linearities as well, this is realized through the ExitFcn command option of fe_simulfe_timeCleanUp (see fe_timeTimeOpt), using '-ExitFcn"nl_spring(''fe_timeCleanUp'')"'. This should be initialized by nl_spring('fe_t
- OutputFcn The output function should be generated by the OutputInit command, since it handles proper interpolation of output as function of the time step, and requires fine tuning in the case of non linear simulations. If nl_spring handles the OutputInit call, OutputFcn is thus reset during initializations. Handling of output time steps using a time vector in OutputFcn is supported.

AssembleCall

The TimeOptAssembleCall must use the -InitFcn callback of fe_caseg Assemble to perform initialization of the non linearities.
Command options are available to tweak the assemble call with minimal user input

- MVR To adapt the assemble call for preassembled reduced models. This typically removes the -load command option of the call as this has to be recovered in the MVR itself.
- skipMKL No to transform the model matrices into mkls objects.
- lumpedMass To adapt the mass matrix mattype to 20 and get a lumped mass matrix.
- compose For more complex calls one can redefine from scratch the assemble call line to which the ad hoc initFcn will be added.

ResidualCall

The TimeOptResidual callback should be a call to mkl_utils, that performs optimized matrix vector products, and the computation of non linear forces handled by nl_functions. Command options allows choosing a call adapted to the type of simulations

- by default a call adapted to the nlnewmark scheme.
- ResidualCallStatic provides a residual adapted to the newton-Raphson schem.
- ResidualCallExplicit provides a residual adapted to the newmark explicit scheme.

fe_timeCleanupCall

The TimeOptFinalCleanupFcn callback must use the -ExitFcn of fe_simulto perform post treatments of non linearities. Custom options classical to the fe_simulFinalCleanup call can be added either in the command string or as a string in second argument.

```
opt.FinalCleanupFcn=nl_spring('fe_timeCleanupCall -cf-1-fullDOF');
% equivalent call with second argument
opt.FinalCleanupFcn=nl_spring('fe_timeCleanupCall','-cf-1-fullDOF');
```

In addition to the standard fe_simulFinalCleanup, the following command options are available (to be specified outside the ExitFcn callback.

- -HDFSave To save the output in a temporary file, and output a v_handle pointer to the saved data. This is useful for RAM optimization matters.
- -HDFfname*fname* In combination to -HDFSave, to specify the file in which the output will be saved.
- -Save To save the output in a temporary file, but keep the results.
- -fname*fname* In combination to -Save, to specify the file in which the output will be saved.

`OutputInitCall`

The `OutputInit` callback is locked for internal `nl_spring` use. Several command options are available that will be forwarded to the `OutputInit` procedure

- `-BlockSave`*N* To initialize a bufferization of the output of size *N*. Results will be saved as blocks containing each *N* saved time steps.
- `-exit` To force exit after initialization. This can be used to check the output format without performing the simulation.
- `-postFcn` To provide a callback that can tweak the output at the end of the `OutputInit` procedure. This can be used for example to initialize `out.Post` post treatments.

`TimeOutputOptions`

Fine tuning of `fe_time` output can be achieved by specifying an `'info','OutputOptions'` case entry.
Accepted fields for the `OutputOptions` structure are

- `.FnlAllT` if defined and equal to 1, non-linear loads are saved at all time steps.
- `.FnlIiplot` if defined and equal to 1, non linear loads are displayed in an iiplot figure as curve `FNL`. If the display timer associated with this figure does not stop automatically, you can stop it with `cingui('TimerStop')`.

`mkl_utils`

Non linearities are treated by `mkl_utils` mex file. Details are provided in `mkl_utils`.

`rheo2NL`

OBSOLETE. Use now `nl_spring NL`.
`NL=nl_spring('rheo2NL',model,DOF,offset);`
This command is used to convert rheological data into a structure of data understandable for `NLforce` command. `DOF` is the list of the DOF coherent with u and v arguments of `NLforce` command. `Offset` is optional. It is a structure of data with fields `.DOF` and `.def` that defined 0 reference for `Fu` and `Fv` tab laws.

`tab`

`model=nl_spring('tab',model);`

nl_spring

This command is used to convert formal rheological description data stored in `model.Stack` to a tabulated law description. The format is likely to change due to optimization of the compiled functionality in `mkl_utils` (see `mkl_utils`).

`BlockSave,BlockLoad`

Undocumented intermediate save of a time block for long simulations that do not fit in memory.

mkl_utils

Purpose

For detailed callback information see `sdtweb('nlspring_timeopt')`.

`Residual`

`Residual` command is used to compute standard residue.
`mkl_utils('residual',r,model,u,v,a,opt,Case);` call modifies variable `r` in memory according to following standard residue computation (implicit Newmark).

```
r = model.K{1}*a + model.K{2}*v + model.K{3}*u - fnl -fc;
```

Typically in `fe_time` computations one has
`opt.Residual='r=-full(fc);mkl_utils(''residual'',r,model,u,v,a,opt,Case);';`
with `fc` the time load (resulting from `DofLoad` entries in model Case) and `fnl` is the sum of the non linear efforts (if any) computed directly by `mkl_utils` (`rotcenter`, `mocirc2`), in the non linear functions (see `sdtweb nl_fun`) or in `nl_spring`. `mkl_utils` then calls the adequate `nl_fun` function (`nl_spring` by default) automatically.

Such call stored in `opt.Residual` is filled by `nl_spring('TimeOpt')` for default simulations.
Model information specifically supported by the residual command are

- `opt.Rayleigh` if the field exists defines a global Rayleigh damping and `opt.Rayleigh(1)*model.K{1` is added to the residual.
- `model.K{2}` can be a data structure describing modal damping with following fields:
  - `.def` : $M\Phi$ vectors as columns.
  - `.data` : c_j modal damping coefficients as a vector. $c_j = 2\omega_j\zeta_j$. A second column has to be set to zero for transient applications.
  - `.type` : `@nl_modaldmp` handling function for callbacks. The following callbacks must be handled
    * matrix projection $tkt = T^T KT$: `tkt=feval(K.type,'getTKT',K,T,Tt,typ)` with `K` the implicit matrix, `T` the right projection matrix, `Tt` the left projection matrix (can be empty or skipped if $Tt = T^T$, `typ` the output type, either *`imp`* to keep the implicit format (by default), or *`full`* to recover a full numeric matrix (to be reserved for small output sizes).
    * vector application $f = Kq$: `f=feval(K.type,'getForce',K,q)` with `K` the implicit matrix, `q` a deformation vector.

- .UseDiag : to be set to one if one wants the output of getTKT to be diagonal (as for a standard dtkt call).
- .K : optional additional damping matrix. This matrix must be in a mkl transposed v_handle format (use v_handle('mklst',K) to convert a matlab matrix to this format). Note that model.K{2}.K is taken in account for the Jacobian computation whereas modal damping is not.
- .defT : the resitution matrix (left side $M\Phi$), that can occur mainly in the case where a non-symmetric projection has been carried out. *E.g.*, the implicit representation of $T_l^T M\Phi \left[\backslash 2\zeta_j\omega_{j\backslash}\right] \Phi^T M T_R$ will use field .def to store $T_R^T M\Phi$ and .defT to store $T_l M\Phi$.

  Corresponding additional residue term is
  $\sum_j [M]\, \phi_j * c_j * \phi_j^T\, [M]^T * v$.

- model.NL can be a stack of non linearities. Column 3 provides a structure with the following standard fields, see nldata.

Typically, fnl is computed by non linearity functions, see nl_fun for details on these functions. The non linear functions are called by mkl_utils to provide the value of fnl at a given state. Two implementations are supported

- An optimized *input-output* formulation, using observation and command matrices c and b documented in nldata. The computation of the observation is possible either on the displacement, the velocity or both, and the command is added to the residual using r = r + b*unl. With unl a vector depending on the observation (c*u, c_v v).
- A used defined addition (older format, that should be only used when the generic b,c format fails to be relevant. In this mode the non linear function must add fnl by itself, choosing the sign convention, using a call of type of_time(-1,fc,fc-fnl);. One will note that the residue vector is named fc in the non linear functions.

chandle

chandle objects are used to streamline communication between mex and MATLAB in iterative processes. They are used in various nl_solve calls and in particular for ModalNemwark and ExpNewmark.

chandle

Purpose

chandle objects are used to streamline communication between mex and MATLAB in iterative processes.
Creation generates a C copy of the matlab array and returns a vhandle.chandle object containing the ID. Register the chandle object for mexAtExit.

- chandle.numType lists currently implemented chandle subtypes.

DiagNewmark

DiagNewmark is an implementation of the Newmark scheme when assuming a fixed diagonal full Jacobian as occurs in modal domain transients (explict or implicit).

ExpNewmark

ExpNewmark is an implementation of the Newmark scheme when assuming a fixed diagonal mass matrix for large explicit dynamic problems.

nl_inout

Support for observation performed in C. .iopt for standard integer options.
.N field : Nunl, (c,1),(c,2),cTrans, (b,2),(b,1),bTrans, Nopt[8],Niopt[9],size(unl,3)[
.opt field ? tc[1] dt0[2] K[3] Fmax[4] Fu functions currently implemented in C are listed under nl_inout fun
The header of the associated class is

```
// nl_inout non linearity 1003
class chandleNl_inout: public chandle {
 public:
  int *irc, *jcc,*irb, *jcb,*iopt;
  double *prc,*pic,*prb,*pib,*unl,*vnl,*snl,*opt;
  int N[11]; // Nunl,  (c,1),(c,2),cTrans,  (b,2),(b,1),bTrans, Nopt[7],Niopt[8],size(u
  __Fu Fu;// (*Fu)(chandleNl_inout*,struct _ROr);
 mxArray* MexData[2];
  chandleNl_inout();
  ~chandleNl_inout();
  void Residual(struct _ROr ROr, double* fc);
  void initCpt(); // Initialize pointers
  void EndStep(); // propagate internal states using StoreType strategy
```

```
};
// Residual structure --------------------------------------------------------
struct _ROr  {
   int    Nk,Nnl;
   double RayleighM,RayleighK,tc;
   double *u,*v,*a,*FNL;
  };
// Default function handle
typedef void(*__Fu)(chandle* ph, struct _ROr ROr);
```

Non linearities list

Purpose

List of supported non linearities. It is possible to create new ones (`sdtweb nl_fun`)

`nl_inout`

`nl_inout` is the more general non linearity, using observation and command matrix associated with elements supporting the kinematics (`cbush` for point connections, see section 1.1.2 , zero thickness volumes for surface connections (where two layers of coincident nodes are considered for a `hexa8` or `penta6` element, see section 1.1.3), volume elements for 3D applications (see section 1.1.4) or the deprecated observation/sensor command/loads as detailed in section 1.1.6 .
The general form of the non-linearity $f_{NL} = b \times f(C.u, C.v)$ is detailed in section 1.1 .
For a list of implemented non-linear constitutive laws xxx
The `pro.NLdata` structure has fields described in section 1.5.1 (with the need to distinguish the form for model declaration and during time integration).
By default, no Jacobian is computed for this non-linearity. Experimental Jacobian are computed according 3 methods according to the `NL.Jacobian` value:

- `0` : no Jacobian. (default).
- `1` : tangent matrices.
- `2` : fixed Jacobian (can be max stiffness / damping or mean, ...).

Then computed matrices are then multiplied by `NL.alphaJK` factor for Jacobian stiffness, and `NL.alphaJD` factor for Jacobian damping.

`nl_contact`

Supports non conform fixed matching contact (squeal applications for example) and (surface contact large displacement, as in rail/wheel interaction for example). For conform meshes, zero thickness elements associated with `p_zt` can be used.
See `p_contact`, `ctc_utils`.

`nl_modaldmp`

Implementation of modal damping. Although modal damping is not a non-linear feature in itself, its implementation requires it to be declared as a non-linearity.
The concept is to provide shapes defined on a part of a model with associated damping ratios. `nl_modaldmp` handles the kinematic projection on the model which can contain superelements. In the case where superelements are used and concerned with modal damping, the shapes provided must be written on the physical DOF of the superelements.

The set of shapes must be stacked in model with a valid ID field. It is a common deformation SDT data structure (see sdtweb def), with an additional .ID field. The .data field is equivalent to the ones of complex modes (see fe_ceig). It is a matrix of two columns respectively giving the frequency and the target damping ratio for each mode.
Since modal damping implies a modal sensor, the features performs both by default. It is however possible to simplify it as a pure modal sensor. The theory around modal sensing/damping can be found in [?].
The pro.NLdata structure has fields

- type: string 'nl_modaldmp'.
- CurveId: the curve ID stacked in model which provides the shapes and their damping ratios.
- SensorOnly: to use the feature only as a modal sensor in a def data structure.

The NLdata structure generation can be integrated using an nl_modaldmp('db') call. See sdtweb nl_spring#setpro for this integration. This is used in transient simulations, and in complex mode computations, see nl_solve.

nl_inout

Purpose

The generic form of `NLdata` specification is discussed in section 1.5 .

`DofSet`

Implementation of linear or large rotation setting of DOF values from time `curves` of a `fe_case` `DofSet` entry. `sdtweb('_eval;','d_fetime.m#NLNewmark_LrDofSet')`.
Supported variants are

- large rotation trajectories (using `MBBryan`) uses `.unl(:,:,1)` corresponding to large displacement and updates `.unl(:,:,j1)` when changing time step to allow velocity computations (currently only valid for fixed time step). Requires setting `.KeepDof=1` and enforcing 6 DOF associated with the master node.
- translation trajectories (xxx).

`Power`

uses `NLdata.opt=[comstr('Power',-32) k n]` and $s_{nl} = k\ u_{nl}^n$.

`FuTable`

uses tabular definitions in either `NLdata.Fu` or `NLdata.Fv`. If you want a case with both simultaneously to be reimplemented please provide a test case. Implements a Jacobian using tangent stiffness. Run example with `sdtweb('_eval;','d_fetime.m#ModalNew_FuTable')`.

`K_t`

Implement time varying matrices and `DofSet` xxx.

`MexIOa`

Implements callback to user defined `.m file` implementations of non-linearities as detailed in section 1.2.3 .

`SCLd`

Large displacement surface contact supported as part of the `contact` module. Possible application : rail/wheel contact. This supports low pass filtering of contact forces, using an evolution equation of the form $\dot{F}_c/\omega_c + F_c = k_c g$. This avoids incorrect bouncing of stiff contacts in implicit computations and ω_c should be a fraction of the sampling frequency (inverse of time step) or the maximum mesh frequency.

`LRFu`

Large rotation tabular spring for multi-body applications. `cbush` kinematics are expected and rotations are assumed using `Bryan` angles in radians. Requires one non-linearity for each spring (not currently vectorized). The `NL.Node` field gives slave and master node for each body (4 nodes). The observed motion is given at the master node and large rotation is used to determine the current position of the slave node. With single axis springs (large rotation rod) the position of the two slave nodes should be disjoint.
The stress vector used by this non-linearity contains 24 elements

- `sx1,sy1,sz1,srx1,sry1,srz1` forces and moments on node 1
- `sx,sy,sz,srx,sry,srz` forces and moments on node 2
- `PX1,PY1,PZ1,rx,ry,rz` absolute position of node 1 and rotations
- `ux,uy,uz` relative displacements in x,y,z directions, `sex,sey,sez` force in element frame.

`slab sensor non-linearity`

This non-linearity supports observation of various quantities during FEM computations. The base definition is associated with the `sdtweb('sensor# scell')`

- For direct resultant sensors in time integration schemes using enforced displacement `-fieldOut4`, leading to `NL.iopt(6+isens)==4`. See more details in xxx

Needs documentation, see `sdtweb d_fetime('slab')`.

```
NL=struct('type','nl_inout','slab',{li(:)},'UserObs',dyn_solve('@doObs'));
mt=stack_set(mt,'pro','Sensors',struct('il',1e5-1,'type','p_null', ...
     'NLdata',NL));
```

Note that in predictor/corrector schemes, it may be necessary to recompute the residual to obtain the correct residual.

`temp still undocumented`

- `FuExpon` $s_{nl} = a\ b^{u_{nl}}$.
- `uMaxw` generic C implementation of `nl_maxwell.m` file
- `FuDahlC` Dahl model with constant force
- `STS` PSA scalar STS

- CLIMA2 connector non-linearity developed with Marco Rosatello
- FuFric basic friction model with $F_k = Ku$ bounded by $\pm F_{max}$.
- nl_bset xxx
- DofSet xxx

Non linearities list (deprecated)

Purpose

nl_maxwell (deprecated)

nl_maxwell describes rheological models using stiffness and damping. *Deprecated implementation that should now be called with an* nl_inout.

.type	'nl_maxwell'
.lab	Label of the non linearity.
.Sens	Observation definition. Cell array of the form {SensType,SensData} where SensType is a string defining the sensor type and SensData a matrix with the sensor data (see sdtweb sensor).
.Load	data structure defining the command as a load (with .DOF and .def fields).
.SE	superelement that defines the rheological model. Only matrices are used (.K field). Mass matrix is ignored. The .DOF field is unused and first DOF are assumed to be the observations defined, and following correspond to internal states.
.NLsteps	Number of sub steps for the integration.
.StoreFNL	strategy to store FNL output.
Ncell	number of cells.

Jacobian is computed using a Guyan condensation keeping only the observation (internal states are condensed) to obtain tangent damping and stiffness.
Internal states are integrated using an independent finite differences explicit scheme, with the same step of time as the main scheme, or a subsampling NL.NLsteps times.
At the first residue computation, the initial internal states are computed according to initial condition in terms of displacements and velocities through a time integration until variation of speed between the 2 last computed steps is lower than opt.RelTol.
Force on the observation DOF (F), displacement (Qc) and velocity (dQc) of the internal DOF, displacement and velocity observations are stored in the NL output.

The command nl_spring db Fu"*type*" is a database of generalized Maxwell rheological models. *type* can be:

- zener standard viscoelastic model. Parameter *k0*, *k1* and *c1* can be given as a string of the form db Fu"zener k0 *k0* k1 *k1* c1 *c1*" in the command.

The example of the standard viscoelastic model is detailed here as an illustration. The standard viscoelastic model, also known as Zener model, is composed by a spring (K_0) in parallel with another spring (K_1) and a serial dashpot (C_1) as displayed figure 3.5.

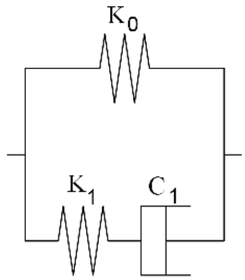


Figure 3.5: Standard viscoelastic model.

In the Laplace domain, the relation between the relative load and the relative displacement is given by

$$F(s) = K(s)X(s) = \frac{K_0K_1 + (K_0 + K_1))C_1s}{K_1 + C_1s} = K_0\frac{1 + s/z}{1 + s/p} \tag{3.2}$$

where p and z are respectively the pole and the zero of the model

$$p = \frac{K_1}{C_1} \tag{3.3}$$

$$z = \frac{K_0K_1}{(K_0 + K_1)\,C_1} \tag{3.4}$$

The maximum loss factor is

$$\eta_m = \frac{p - z}{2\sqrt{pz}} = \frac{1}{2}\frac{K_1}{\sqrt{K_0\,(K_0 + K_1)}} \tag{3.5}$$

and obtained for pulsation

$$\omega_m = \sqrt{pz} = \frac{K_1}{C_1}\sqrt{\frac{K_0}{K_0 + K_1}} \tag{3.6}$$

K_0 is the static stiffness of the model. Typically $K_1 = \frac{K_0}{2}$ and C_1 is defined so that the damping is maximal for the frequency of interest.
Following example considers $K_0 = 1000N/m$, $K_1 = 500N/m$ and $C_1 = 1.4Ns/m$. These parameters lead to a maximum loss factor of 20.14% for a frequency of 46.41Hz. The module and the loss factor are represented in figure 3.6.

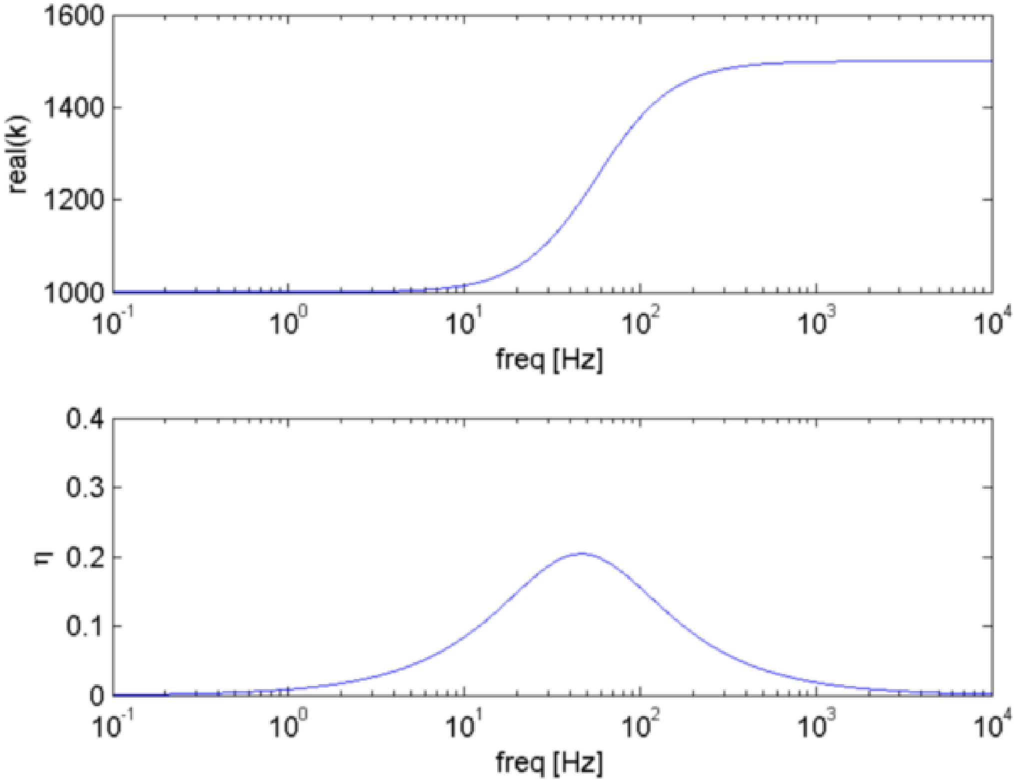


Figure 3.6: Module and loss factor.

Following example consists in a mass of 1e-2kg linked to the ground by the Zener model. Initial displacement corresponding to a 1N load on the mass is imposed and then a time simulation is performed.

```
% parameters
 param.m=1e-2; param.dt=1e-4; param.N=1e3;
 param.k0=1e3; param.k1=param.k0/2; param.c1=1.4; % zener parameters
% define model
model=struct('Node',[1 0 0 0 0 0 0],...
           'Elt',[Inf abs('mass1') 0; 1 0 0 param.m  0 0 0]);
% define nl_maxwell data
data=nl_maxwell(sprintf('db Fu"zener k0 %.15g  k1 %.15g c1 %.15g"',....
                         param.k0,param.k1,param.c1));
data.Sens{2}=1.03; % translation sensor defining nl_maxwell inputs
% define associated property
r1=p_spring('default'); r1=feutil('rmfield',r1,'name');
```

```
r1.NLdata=data; r1.il(3)=param.k0;
r1.il(1)=100; model=stack_set(model,'pro','zener',r1);
% define option for time integration
opt=d_fetime('TimeOpt');
opt.NeedUVA=[1 1 1];
opt.Follow=1; opt.RelTol=-1e-5;
opt.Opt(7)=-1; % factor type sparse
opt.Opt(4)=param.dt; opt.Opt(5)=param.N; % NSteps
%opt.IterEnd='eval(opt.Residual)'; % to compute real FNL for current state
% Initial state
r1=data.SE.K{3}\[1;0]; r1=r1(1); % initial displacement for 1N load
model=stack_set(model,'curve','q0',struct('def',r1,'DOF',1.03));
% Time computation
def0=fe_time(opt,model); ci=iiplot; % compute

% The same but NL as a model
SE2=data.SE;
SE2.Elt(end+1:end+2,1:6)=[Inf abs('mass1'); 1 0 0 param.m 0 0];
SE2=fe_caseg('assemble -secdof -matdes 2 3 1 -reset',SE2);
r1=SE2.K{3}\[1;0]; %r1=r1(1);
SE2=stack_set(SE2,'curve','q0',struct('def',r1,'DOF',SE2.DOF));
def20=fe_time(opt,SE2); % compute
F20=SE2.K{2}*def20.v+SE2.K{3}*def20.def; F20=F20(1,:);
% zener labs: {'zener-F1','zener-q1','zener-q1-1','zener-dq1','zener-dq1-1'}
NL20=struct('X',{{def20.data {'LIN-F1';'LIN-Qc1';'LIN-dQc1';'LIN-unl1';'LIN-vnl1';'ft'}}
 'Xlab',{fe_curve('datatypecell','time')},...
 'Y',[F20' (fe_c(def20.DOF,1.03)*def20.def)'...
     (fe_c(def20.DOF,3.03)*def20.def)'...
     (fe_c(def20.DOF,1.03)*def20.v)'...
     (fe_c(def20.DOF,3.03)*def20.v)'    zeros(size(def20.def,2),1)    ]);
NL20.name='NLfromLIN';
iicom('curveinit',{'curve','NL(1)',ci.Stack{'NL(1)'};
        'curve',NL20.name,NL20});
A=ci.Stack{'NL(1)'}.Y(2:end,:);B=NL20.Y(2:end,:);t=NL20.X{1}(2:end);i2=any(A);
if norm(A(:,i2)-B(:,i2),'inf')/norm(B,'inf')>0.01
 figure(1);plot(t,A,'--o',t,B,'-')
 sdtw('_err','something has changed')
end
```

Non linearities list (deprecated)

DofKuva

DofKuva defines a non linear load of the form $av^e_{Dof}\,[K]\,\{V\}$ with a scalar coefficient a, a scalar v_{Dof} extracted from displacement, velocity or acceleration, and V a field specified as follows

.type	'DofKuva'
.lab	Label of the non linearity.
.Dof	Dof of Case.DOF.
.Dofuva	[1 0 0] for displacement Dof, [0 1 0] for velocity and [0 0 1] for acceleration.
.MatTyp	Type of the matrix K (see MatType). Desired matrix is automatically assembled before time computation.
.factor	Scalar factor a.
.exponent	Exponent of the DOF.
.uva	Type of vector V: [1 0 0] for displacement, [0 1 0] for velocity and [0 0 1] for acceleration.

For example one can take in account gyroscopic effect in a time computation with a NL of the form

```
model=stack_set(model,'pro','DofKuva1005', ... % gyroscopic effects
   struct('il',[1005 fe_mat('p_spring','SI',1) 0 0 0 0 0],...
   'type','p_spring','NLdata',struct(...
       'type','DofKuva','lab','gyroscopic effect', ...
       'Dof',1.06,'Dofuva',[0 1 0],'MatTyp',7,...
       'factor',-1,'exponent',1,'uva',[0 1 0])));
```

DofV

DofV defines a non linear effort of the following form (product of a fixed vector and a dof) $(u)^{exponent}.V$ NDdata fields for this non-linearity are

.type	'DofV'
.lab	Label of the non linearity.
.Dof	Dof of Case.DOF.
.Dofuva	[1 0 0] for displacement Dof, [0 1 0] for velocity and [0 0 1] for acceleration.
.exponent	Exponent of the DOF.
.def	data structure with fields .def which defines vector V and .DOF which defines corresponding DOF.

nl_spring

nl_spring defines a non linear load from rheological information (stop, tabulated damping or stiffness laws etc.) between 2 DOF.
To define a non linear spring, one has to add a classic celas element, linear spring between only

2 DOF. The non linear aspect is described by associated properties as a 'pro' entry in the model Stack.
One can describe non linearity by a formal rheological description using one or more of following fields in the pro Stack entry:

- .But : [dumax k0 c0 dumin k1 c1] bumpstop. For du from dumin to dumax, f=0. For du>dumax, k0 stiffness is applied to du-dumax, and for du<dumin, k1 stiffness is applied to du-dumin. Damping is not taken in account at this time (due to tabulated law strategy).
- .Fsec : [fsec,cpenal]. For dv<-fsec/cpenal or for dv>fsec/cpenal, f=fsec is applied. For -fsec/cpenal<dv<fsec/cpenal, f=cpenal*dv is applied. If omitted, cpenal=1e5.
- .K
- .C

This information will be converted in tabulated laws Fu and Fv using nl_spring tab (low level call that should be automatically called at the beginning of time computation).
One can also describe non linearity with a tabulated effort / relative displacement and effort / relative velocity law between the DOF (dof2-dof1), respectively in the Fu and Fv fields of the pro Stack entry. First column of Fu (resp. Fv) gives the relative displacements (resp. velocities) and second column gives the efforts. One can give a coefficient av factor of Fv depending on relative displacement as a third column of Fu. It can be useful to describe a non linearity depending on relative displacement and relative velocity. Force applied is F=av(du).Fv(dv). It is used in particular to describe damping in a stop (.But NL).
Following example performs a non linear time computation on a simple 2-node model:

```
RT=struct('nmap',containers.Map);
% sdtweb d_fetime Mesh2DOF        % For meshing scrip
% sdtweb d_fetime LegacyBumpStop  % For NLdata definition
RT.nmap('Transient')='out=fe_time(mo1);out=stack_set(mo1,''curve'',''RunResult'',out);'
li={'MeshCfg{"d_fetime(2DOF):LegacyBumpStop{Z0}"}', ':' ...
      'SimuCfg{Imp{1m,10,uva110}}',':', ... % implicit NLNewmark simulation
      'RunCfg{Transient}'};disp(comstr(li(1:2:end)',-30))
model=sdtm.range(RT,horzcat(li{:}));def=sdth.urn('RunResult',model);

li{3}='SimuCfg{Exp{1m,10,uva110}}'; % Same using explicit
mo2=sdtm.range(RT,horzcat(li{:}));d2=sdth.urn('RunResult',mo2);

figure(10);clf;% plot some comparison between results
subplot(211);plot(def.data,[def.def' d2.def']);xlabel('Time [s]');ylabel('displacement'
```

```
subplot(212);plot(def.data,[def.v' d2.v']);xlabel('Time [s]');ylabel('velocity')
legend('Implicit','Explicit');setlines;
sdth.os(10,'@OsDic',{'ImGrid','ImSw80','ImTight'})
```

Following example deals with a clamped-free beam, with a bilateral bump stop at the free end.

```
% define model:
L=1; b=1e-2; h=2e-2; e=1e-3; % dimensions
model=[];
model.Node=[1  0 0 0  0 0 0; 2  0 0 0  L 0 0];
model.Elt=[Inf abs('celas') 0 0;
           2 0  2 0     100  1 110 0;   % linear celas
           Inf abs('beam1') 0 0;
           1 2  1 1    0 1 0 0
           ];
model=feutil(sprintf('RefineBeam %.15g',L/20),model);
model=fe_case(model,'FixDof','base',1); % clamps 1st end
model=fe_case(model,'FixDof','2D',[0.03;0.04;0.05]); % 2D motion
% model properties:
model.pl=m_elastic('dbval 1 steel');
model.il=p_beam(sprintf('dbval 1 BOX %.15g %.15g %.15g %.15g',b,h,e,e));
% Bump stop NL:
model=stack_set(model,'pro','celas1',...
        struct('il',[100 fe_mat('p_spring','SI',1) 1e-9 0 0 0 0],...
               'type','p_spring',...
               'NLdata',struct('type','nl_inout',...
                               'but',[0.02 5e2 0 -0.02 5e2 0],... % gap knl cnl...
                               'umin',3)));
if 1==1
 model=fe_case(model,'DofLoad','in',struct('DOF',2.02,'def',50));
 model=fe_curve(model,'set','input','TestStep t1=0.02');
else
 f=linspace(12,18,3);
 model=fe_case(model,'DofLoad','in',struct('DOF',2.02,'def',1));
 model=fe_curve(model,'set','input',sprintf('Testeval cos(%.15g*t)',f(1)*2*pi));
end
model=fe_case(model,'setcurve','in','input');
% Time computation:
opt=d_fetime('TimeOpt dt=1e-3 tend=10'); opt.NeedUVA=[1 1 0];
def=fe_time(opt,model);
```

RotCenter

The Rotcenter joint is used to introduce a penalized translation link between two nodes A and B (rotation DOFs of NL entry are ignored), where the motion of A is defined in a rotating frame associated with angle θ_A and large angle rotation $R_{LG}(\theta_A)$. The indices G and L are used to indicate vectors in global and local coordinates respectively.
The positions of nodes are given by

$$\begin{array}{rcl} \{x_A\}_G & = & [R_{GL}]\left(\{p_A\}+\{u_A\}_L\right) \\ \{x_B\}_G & = & \left(\{p_B\}+\{u_B\}_G\right) \end{array} \tag{3.7}$$

which leads to expressions of the loads as

$$\begin{array}{rcl} \{F_A\}_L & = & [R_{LG}]\left(K\left(\{x_B\}_G-\{x_A\}_G\right)\right) \\ \{F_B\}_G & = & K\left(\{x_A\}_G-\{x_B\}_G\right) \end{array} \tag{3.8}$$

To account for viscous damping loads in the joints, one must also compute velocities. Using (**??**), one obtains

$$\begin{array}{rcl} \{\dot{x}_A\}_G & = & [R_{GL}]\left(\{\dot{u}_A\}_L+\{\omega(t)\}\wedge\{p_A+u_A\}_L\right) \\ \{\dot{x}_B\}_G & = & \{\dot{u}_B\}_G \end{array} \tag{3.9}$$

Velocity computations are currently incorrect with u_A ignored in the rotation effect. So that viscous damping loads can be added

$$\begin{array}{rcl} \{FC_A\}_L & = & [R_{LG}]\left(K\left(\{\dot{x}_B\}_G-\{\dot{x}_A\}_G\right)\right) \\ \{FC_B\}_G & = & K\left(\{\dot{x}_A\}_G-\{\dot{x}_B\}_G\right) \end{array} \tag{3.10}$$

For a linearization around a given state (needed for frequency domain computations or building a sensor observation matrix),

$$\left\{\begin{array}{c} q_{AG} \\ q_{BG} \end{array}\right\} = \left[\begin{array}{cc} R_{GL} & 0 \\ 0 & I \end{array}\right]\left\{\begin{array}{c} q_{AL} \\ q_{BG} \end{array}\right\} \tag{3.11}$$

In global basis, stiffness matrix of a celas link is given by

$$k\left[\begin{array}{cc} I & -I \\ -I & I \end{array}\right] \tag{3.12}$$

which leads to the following stiffness matrix

$$\left[\begin{array}{cc} R_{GL}^T & 0 \\ 0 & I \end{array}\right] k \left[\begin{array}{cc} I & -I \\ -I & I \end{array}\right]\left[\begin{array}{cc} R_{GL} & 0 \\ 0 & I \end{array}\right] = k\left[\begin{array}{cc} I & -R_{GL}^T \\ -R_{GL} & I \end{array}\right] \tag{3.13}$$

where q_A DOFs are in the local basis (motion relative to the shaft in its initial position) and q_B are in the global frame.

`data` describing this link is stored in model stack as a `p_spring` pro entry. Stiffness and damping are stored respectively as 3rd and 5th column of the `data.il` field (standard linear spring, see `sdtweb('p_spring')`).
`NDdata` fields:

- `.type` string `'RotCenter'`.
- `.sel` a `FindElt` command to find `celas` of `RotCenter` type.
- `.k` this field should not be used. `.JCoef` field should be used instead and has priority. Stiffness used for Jacobian computation. Damping is not taken in account in Jacobian in this case.
- `.JCoef` coefficient of celas stiffness and damping for Jacobian computation. Default is 1.
- `.drot` the rotation DOF.
- `.lab` label.

`nl_rotCenter`

This non linearity can be used to connect 2 points A and B, where the motion of A is defined in a rotating frame associated with angle θ_A and large angle rotation $R_{LG}(\theta_A)$. More generally A and B are no real nodes but defined implicitly as observation matrices. `nl_rotcenter` is an extension of `RotCenter` documented above, using observation matrices which is more general.

- `.type` string `'nl_rotcenter'`.
- `.sel` a `FindElt` command to find elements associated to the NL link
- `.JCoef` coefficient of celas stiffness and damping for Jacobian computation. Default is 1.
- `.drot` the rotation DOF.
- `.lab` label.
- `.Weights` (optional) Weight of the stiffness in a pivot link (in fact computed force is multiplied by the weight factors before being applied so that the sum of weight coef divided by number of points by pivot should be equal to 1).
- `.Stack` Stack of cta coupling. Of the form `{'cta', 'name',{r1,r2}}`, where `'cta'` is a constant string defining the type of the link, `'name'` a string containing the name of corresponding links. `r1` is the observation in the first (rotating) part. It is a data structure with fields `.Node` defining the nodes involved, `.cta` defining the observation matrix, `.DOF` defining corresponding DOF (as many columns as in `.cta`) and `.SeName` defining as a string the name of the superelement where cta is defined (if omitted, it is assumed that DOF and cta are defined on the model.DOF - no superelement -). `r2` is the same for the non rotating part.

An example can be found in `t_nlspring 2beam`.
Default uses the damping and stiffness defined in the il field of the `p_spring` pro entry to model a linear spring/damper between the 2 parts (stiffness il(3) and damping il(5)).

Defining a `xb` parameter, the Excite NONL law will be applied instead of the spring/damper. Parameter that are to be defined are

- `.xb` Radial clearance.
- `.kb` Stiffness at radial clearance.
- `.cb` Damping at radial clearance.

Stiffness and damping at initial position are given in corresponding `p_spring` properties il(3) and il(5). For example:

```
cf.mdl=nl_spring('setpro ProId 103 k 371 c 2000e-3 xb 0.03 kb 37100 cb 5',cf.mdl);
```

`rod1`

The `rod1` non-linear connection is a simple penalized rigid link. One considers two nodes A and B (see figure 3.7).

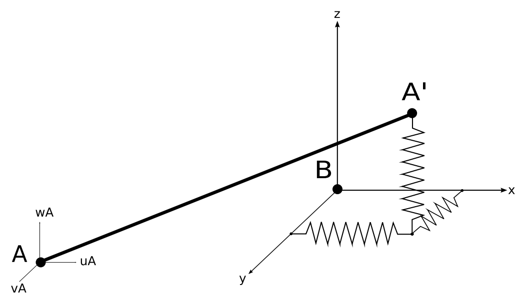


Figure 3.7: Large rotation rod functional representation.

Currently, one can introduce masses at points A and B. `mass2` elements should be used to account for the actual position of the center of gravity.
The global non linear load associated with the rod is thus

$$F_{rod} \quad = \quad k_r\left(\|\{x_B - x_A\}\| - L_0\right)\frac{\{x_B - x_A\}}{\|\{x_B - x_A\}\|} \tag{3.14}$$

which accounts for a load proportional to the length fluctuation around L_0 (penalized rod model).

Non linearities list (deprecated)

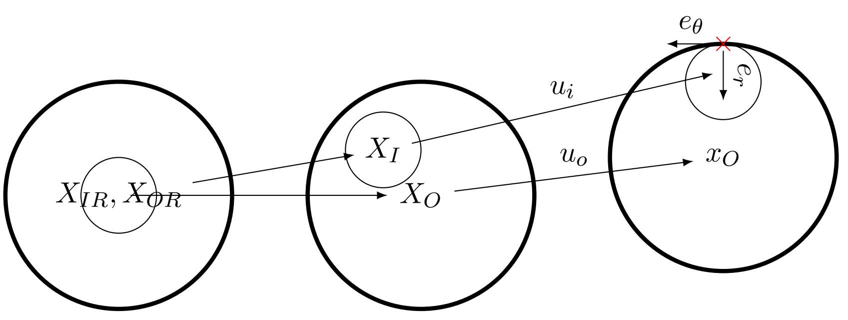
When linearizing, one considers a strain energy given by $k_r \|q_B - q_{A'}\|^2$ with the motion at node A' being related to the 6 DOFs at node A by

$$\{q_{A'}\} = \begin{bmatrix} I & \left[\vec{AB}\wedge\right] \\ 0 & I \end{bmatrix} \{q_A\} \tag{3.15}$$

Node A node is free to rotate. The linearized stiffness thus corresponds to an axial stiffness in the direction of the rod. The computation of the stiffness is however based on the current position of the extremity nodes, a difficulty in model manipulations is thus to translate these nodes.
`data` describing this link is stored in model stack as a `p_spring` pro entry. Stiffness and damping are stored respectively as 3rd and 5th column of the `data.il` field (standard linear spring, see `sdtweb('p_spring')`). NL information is stored in the `data.NLdata` field which has itself following fields :

- type : string `'rod1'`.
- sel : a `FindElt` command to find associated `celas` of `rod1` type ((`'proid100'`).
- ulim : build tabulated law from -ulim to ulim. Default is 1e3.
- lab : label.

`nl_gapcyl`



This non-linearity implements non-linear contact between two cylinders of radius R_O for the outer cylinder and R_I for the inner cylinder with motion defined on the cylinder center line. Assuming the mesh to be defined in an updated Lagrangian configuration where the center lines X_I and X_O at not coincident, the positions in a deformed states are given by $x_I = X_I + u_I$ and $x_O = X_O + u_O$. Contact may only occur when the cylinders are not centered. When the two cylinders are not centered the non-linear observation is given by

$$u_{NL} = x_O - x_I = u_O - u_I + (X_O - X_I) = [c]\{q\} + u_{NL0} \tag{3.16}$$

From this distance between the center lines, a cylindrical basis is defined with $e_r(q)$ along the direction from x_I to x_O and e_θ forming a direct basis with the cylinder axis (kept constant from the initial value of the updated lagrangian position).
The functional definition of the contact force uses the gap in the e_r direction defined by

$$g = \{e_r\}^T \{u_{NL}\} - (R_O - R_I) \tag{3.17}$$

where $R_O - R_I = d$ is stored as parameter NLdata.d and the contact leads to two opposite forces on the cylinder center lines

$$\{f_I\} = \{-f_O\} = f(g)\{e_r\} \tag{3.18}$$

The current implementation assumes rotations to be small enough to ignore the difference between e_r and the corresponding vector E_r in the upated Lagrangian configuration.
If update of mesh leads to lateral slip, then u_I may account for longitudinal position of the contact point along the beam using shapes functions.
When linearizing the contact around a given point, the stiffness $\partial f \partial g$ only occurs in the e_r direction.

Creating a new non linearity: nl_fun.m

Purpose

The structure of nl_spring allows creating any new non-linearity through the use of a dedicated function, named nl_fun.m. This function which non-linearity name will be fun, will be automatically called by nl_spring for classical operations.

The function structure has been designed to comply with specific needs. Standard calls have been defined, which are detailed below:

- **Residue computation**, called by mkl_utils (sdtweb mkl_utils), must output the entry force minus the non linear force computed. The call performed is

  ```
  nl_fun(r2,fc,model,u,v,a,opt,Case)
  ```

  This call is low level and must modify fc using sp_util('setinput') as fc-fnl where fnl is the non linear force computed. Note that this is the only possible call for nargin==8. Note that mkl_utils allows a formalism with precomputed observations, using fields unl.

- **Jacobian computation**, must output the tangent stiffness and tangent damping matrices associated to the non linearity. The call performed is

  ```
  [kj2,cj2]=nl_fun(NL,[],model,u,v,[],opt,Case,RunOpt);
  ```

  This call must output either empty matrices if no tangent nor Jacobian matrix is associated to the non linearity, or matrices expressed on the DOF vector of Case.DOF. The first matrix is the tangent stiffness matrix, the second one is the tangent damping matrix. Typically there are 3 normalized methods to be defined (but not all of them must be defined, and more can be defined) according to the NL.Jacobian value:

  - 0 : no Jacobian. (default).
  - 1 : tangent matrices.
  - 2 : fixed Jacobian (can be max stiffness / damping or mean, ...).

  Then computed matrices are then multiplied by NL.alphaJK factor for Jacobian stiffness, and NL.alphaJD factor for Jacobian damping.

- **Initializations** for fe_time, must initialize the model non-linearity for non linear forces computation

  The call must generate the non linearity stored in model.NL, it can optionally generate non linear DOF and labels. The call performed is of the type.

  ```
  NL=nl_fun('init',data,mo1);
  ```

NL is a struct containing at least the field type with the nl_fun handle (*e.g.* NL.type=@nl_fun). data contains the Stack,pro entry, and mo1 is the model, named mo1 where the call is performed.

- **ParamEdit** returns the ParamEdit string allowing integrated parameters interpretation (for internal SDT use).

  The call performed is of the type.

  ```
  st=nl_fun('ParamEdit');
  ```

- **db** returns default NLdata fields for a non linearity. This allows integrated building of non-linearities in a model. This function can call ParamEdit to allow interactive setup.

  This call must return a NLdata field and is of the type

  ```
  NLdata=nl_fun('db data 0');
  ```

- **Energy post treatments** capability, should return the elastic energy stored in the non-linearity as a vector with as many lines as time steps in the output.

  The call performed by nl_solvePost is of the form

  ```
  r2 = nl_fun('PostEnerNL');
  ```

  The non linearity function can access in caller fields RO, out, model, NL, i1 with

  - RO a structure with fields EnerP and EnerK respectively containing the potential and kinetic energy.
  - out the fe_timeoutput.
  - model the model used in the simulation.
  - NL the NL containing the data of the non-linearity called.
  - i1 the row index of out.Post that is currently generated.

- **Renumbering** capability, must return the non-linearity written for the new renumbered nodes, elements, dof, ...

  The call performed (by feutilbfor example) is of the type

  ```
  NL=nl_fun('renumber',NL,nind);
  ```

  nind is the renumbering vector.

The designed nl_fun template is given in the non-linear toolbox, sdtweb nl_fun.m#1. It is a functional non linear function, computing a zero non linear force. The definition of a non linearity using nl_fun in a standard SDT model is given in the following.

Creating a new non linearity: nl_fun.m

```
% A standard SDT model
model=struct('Node',[1  0 0 0  0 0 0; 2  0 0 0  0 0 1],...
             'Elt',[Inf abs('celas') 0 0;
             1 2 3 -3  0 1 0  10;   % linear celas
            ]);
% Define a non linearity of type nl_fun
model=nl_spring('SetPro ProId 100',model,nl_fun('db data0'));
%Equivalent to
% model=stack_set(model,'pro','nl_fun',...
% struct('il',[100 fe_mat('p_spring','SI',1)],...
%        'type','p_spring',...
%        'NLdata',struct('type','nl_fun','data',[])));

% Define the case
model=fe_case(model,'FixDof','base',1);
model=fe_case(model,'DofLoad','in',struct('DOF',2.03,'def',1));
model=fe_curve(model,'set','input','TestStep t1=0.02');
model=fe_case(model,'setcurve','in','input');
% Define the TimeOpt and compute the solution
opt=nl_solve('TimeOpt'); opt.Opt([4 5])=[1e-3 1e4];opt.NeedUVA=[1 1 0];
def=fe_time(opt,model);
```

nl_solve

Purpose

Integrated non linear simulations

Description

The simulation of non linearities require special handling in SDT, which is packaged in the non linear toolbox. This function aims at performing classical studies, such as done by fe_simulfor classical SDT models with this special handling.
See nllist for the list of supported non linearities.

TimeOpt

nl_solve('TimeOptMethod',RO) used to initialize fe_time options for later simulation. Currently implemented methods

- Explicit Newmark scheme
- Stat non-linear static Newton
- Theta method time integration.
- ModalNewmark uses an optimized fully C based integration for the case where DOF correspond to modal degree of freedom. The stepped sine strategy is discussed in section 2.3.2 .
- NLNewmark default implicit Newmark scheme.

Associated options provided in RO or in the command are

- .tend end time of simulation. Used to initialize .ts=ceil(tend/dt).

Static

To compute the static state of a model with non-linearities.

```
q0=nl_solve('static',model);
```

It is possible to use custom fe_time simulation properties using the model stack entry info,TimeOptStat. See nl_spring TimeOpt for fields and defaults.
It is possible to use as command option any field from the usual static simulation option, see sdtweb nl_spring#TimeOpt to have more details. *E.g.* To redefine on the fly the maximum number of iteration, one can enter [q0,opt]=nl_solve('static maxiter 100',model);.
By default, the staticNewton algorithm implemented in fe_time is called.
An Uzawa algorithm is also implemented in nl_solve, under the method static nl_solve uzawa. This algorithm is very different from the staticNewton one since here the solution is not incremented

but fully re-computed at each iteration. This is useful when some non-linear forces do not derive from potentials. Command `StaticUzawa` can be used in `nl_solve` – to access it: `q0=nl_solve('static Uzawa',model);`.

`Mode`

The definition of modes for non-linear models is not straight forward. This command aims at computing tangent modes as function of a non-linear model current state. The resolution thus concerns a linear model with tangent stiffness, damping matrices corresponding to the model current displacement, velocity, acceleration state. The eigenvalue solvers used are then `fe_eig`for real modes and `fe_ceig`for complex modes.
By default, modes tangent to a static state are computed. A static simulation is performed to produce a model state from which tangent matrices are computed. It is also possible to compute tangent modes at specific instants during a transient simulation, at `SaveTimes` instant, and to store frequency/damping data and deformations.
A set of command options allows detailing the mode computation wanted and the output.
Accepted command options to control the model computation itself are

- `-allmatdes` to ask for an assembly with all matrix types assembled, the default assembly command used is `-matdes 2 3 1`. This command can be used to keep specific matrix types defined in pre-assembled superelements.
- `cpx` for complex mode computation (default is real mode computation).
- `-evalFNL` (in combination with command `traj`) asks to recompute the `FNL` field on the fly based on displacements prior to mode computation. This command is useful when solutions used for the tangent state have been imported from an external solver.
- `skip` skips `fe_time`simulations and performs the complex mode computation based on the zero deformation and with initialized values of non linearities. The behavior will thus depend on the non linearity initialization strategy. *E.g.* for contact see (`p_contact`), the `-skip` option will consider a full contact state.
- `stat` for mode computation based on a static state (typically after a `fe_time staticNewton` simulation). Uses model stack entry `info,TimeOptStat`.
- `time` for mode computations during a transient simulation (exclusive with the default `-stat` option). Uses model stack entry `info,TimeOpt`.
- `traj` for mode computations based on states provided as an additional argument.

The `-stat` and `-time` options are mutually exclusive and define the base solver options to be used by `fe_time`for the preliminary state computation. With `-stat` option (default) the stack entry `info,`

TimeOptStat will be sought and used if found. With -time option, the stack entry info,TimeOpt will be used if found.
The -traj option is complementary and is used to force the complex mode computation on provided states. On can either provide the state in deformation curve format, see sdtweb def as a last argument, or use predefined stack entries. In -stat mode (default), the model stack entry curve,q0 will be sought and used if found, if not the result will use the -skip mode. In -time mode, the model stack entry curve,TSIM will be sought and used. If not found an error will occur.
Accepted command options to control the output format are

- -addedOnly (in combination with backTgtMdl) only outputs the tangent matrices as a superelement that would have been added to the base matrices for the mode computation.
- -alpha (requires -cpx) to also output the real mode participation to the complex modes. This is in fact the projection of the complex modes on the real mode basis.
- -backTgtMdl outputs the tangent model that would have been used for mode computation.
- -dataOnly to save only the frequency, damping data (does not store the deformation field). The output is then under a frequency tracking curve in the iiplotformat.
- -fullDOF to output the deformation fields restituted on the unconstrained DOF.
- -keepT*val* (requires -cpx) to allow keeping the underlying real mode basis when computing complex modes. With *val* set to *1*, the initial real mode basis will be kept under field def.T, as an additional independent output, coherent with the -alpha command option. With *val* set to *2*, the complex modes will not be restituted but expressed on the subspace used for their computation, the subspace basis will be output in def.Mode.TR, allowing a complete compatibility with feploton-the-fly restitution strategy for display. This latter option is the most complete and efficient strategy. Complete subspace information is kept and can be used for further exploitation, complex mode projection on real mode (-alpha) is naturally obtained, and memory footprint is optimized as the storage size of the subspace is commonly lower by a factor 1.5 to 2 than the complex mode basis;
- -noPost is used to skip any solution post treatment, and outputs the raw mode structure straight from the solver.
- -PostFcn''*cam*'' is used to perform specific post-treatments on the mode output after computation.
- -real "ModeBas" (requires -cpx) to specify a particular real mode basis on which the complex modes will be computed. The real mode basis is supposed to be stored in the model stack entry curve, ModeBas.

Internally, the solver defines and uses the model stack entry `info,SolveOpt` structure to handle the options documented above. One can define it as a structure with the fields documented (case sensitive) and provide it instead of the `EigOpt` input. Additional advanced field are then accessible

- `EigOpt` a vector providing eigenvalue computation options following the `fe_eig`format.
- `cpx''command''` to externalize the mode computation. This command is by default a Boolean telling the solver whether to perform a complex mode computation (set to 1) or a real mode computation (set to 0). If a string is provided, the solver will evaluate it as an external command instead of performing mode computation. One then gets access to the `nl_solve` mode computation framework for ones' own solver.
- `ind` provides a vector of indices that will be used to restrict the output to the indexed modes.
- `SubDef` provides a command that will be evaluated to perform a dynamic user defined restriction to the output modes, it is thus more general than the `ind` option. The result of the command has to be a vector of indices.
- `AssembleCall` to force a specific `AssembleCall` strategy.

The various input and output strategies allow for the support of several input syntax. The following calls are thus accepted, with `model` a standard SDT model, `Case` a standard SDT case structure, `eigopt` either a vector providing options for `fe_eig`or a structure with optional fields defined above, `def` a standard SDT deformation field structure used by `-traj` when necessary.

```
nl_solve('mode',model);
nl_solve('mode',model,eigopt);
nl_solve('mode',model,Case,eigopt);
nl_solve('mode',model,def);
nl_solve('mode',model,Case,def);
nl_solve('mode',model,eigopt,def);
nl_solve('mode',model,Case,eigopt,def);
```

Sample calls using command options to extract tangent modes are given below.

```
def0=nl_solve('Mode',model)
def0=nl_solve('Mode',model,[5 20 1e3]) % with eigopt
def0=nl_solve('Mode-stat-fullDOF',model);
defT=nl_solve('Mode-time',model);
hist=nl_solve('Mode-time-dataOnly',model);
histC=nl_solve('Mode-cpx-time-dataOnly',model);
defC=nl_solve('Mode-cpx-time-alpha-real''MyBas''-fullDOF',model);
def1=nl_solve('Mode-skip-fullDOF',model);
```

Post

The Post command allows performing energy and potential further post treatments of a non-linear simulation. The output is integrated in the standard fe_timesimulation outputs in field out.Post that is a three columns cell array directly compatible with the iiplot format.
To obtain the post treatments, one must define them prior to starting the simulation. Direct computation of the post-treatments *a posteriori* is also possible.

- Command PostDefine adapts the TimeOpt structure to initialize fields in the output and trigger post treatments in the final cleanup phase. The PostDefine call must thus be performed after the TimeOpt call. Using this command itself prior to a time simulation is enough to obtain the post treatments.

```
opt=nl_solve('PostDefine keys',opt); % adapts the opt structure.
model=nl_solve('PostDefine keys',model); % adapts the opt structure contained in mo
```

- Command PostLab provides the list of available post treatment keywords. The input is a structure with fields the post treatment keywords and a logical.
- Command PostHist provides an iiplot curve structure adapted to the post treatments. On can provide a PostLab structure with fields assigned to 1 for desired posts to obtain the corresponding curve.
- Command PostCompute computes the post treatments and store them in out.Post. This command is internally called if the PostDefine command was used prior to the time simulation. For *a posteriori* computations, the user must provide the out as a standard fe_time format initialized with Post field and the assembled model. The model must feature a stack entry info, OutputOptions with field Post containing the PostLab structure.

```
% Generate a TimeOpt
opt=nl_solve('TimeOpt');
Perform the time simulation
def=fe_time(opt,model);
% Initialize for post treatments
[def,RO]=nl_solve('PostInit EnerM',def);
model=stack_set(model,'info','OutputOptions',...
struct('Post',RO));
% Assemble model with non linearities
model=fe_case(opt.AssembleCall,model);
% Compute post treatments
def=nl_solve('PostCompute',def,model);
% display in iiplot
iiplot(def.Post);
```

- Command `PostInit` is an internal function that initializes the output `Post` field at the start of the simulation. Early initialization is useful if the post treatments are performed on the fly by the `OutputFcn`.

The following post treatments are available

- `EnerP` The linear potential, or strain energy.
- `EnerK` The kinetic energy.
- `EnerNL` The elastic or strain energy stored in the non linearities.
- `EnerM` The mechanical energy, defined as `EnerP + EnerK + EnerNL`.
- `PDiss` The instant dissipated power.
- `EnerDiss` The cumulated dissipated energy over time.

Command `PostEstimate` allows analyzing the energy curves to compute

- `Fest` an estimation of the vibration frequency (based on quasi-sinusoidal oscillations)
- `DmpR` an estimation of the damping ratio based on the estimated frequency by computing the dissipated mechanical energy. $\zeta = \frac{1}{4\pi} \log \frac{E_m(t_0)}{E_m(t_1)}$
- `Emax` the maximum mechanical energy identified on the cycle analyzed.
- `EDiss` the dissipated mechanical energy over the cycle analyzed.

The following command options allow altering the estimation

- `-cf`*i* to specify the iiplot figure with handle *i*.
- `-bandpass`*fmax* to perform a bandpass from 0 to *fmax* `Hz` filtering prior to the analysis.
- `-curveName''`*name*`''` to provide the `iiplot` stacked curve name to exploit.
- `-baseOn''`*name*`''` to specify on which post treated curve the frequency estimation is made.
- `-globalMaxTol`*val* to provide a relative tolerance over which a point is detected as close to the global maximum. This is exploited to detect the peaks over the energy signal analyzed.
- `-localMax` to estimate the frequency by detecting the zeros of the signal derivative (less robust).
- `-unit''`*II*`''` to provide an output unit system.

- XFcn''*str*'' to provide a function call to be evaluated that can perform further post treatments(e.g. model specific posts). The called function can access out, outLab, st, j1 with out a matrix containing the output with as many lines as provided curves and as many columns as outputs data, outLab a cell array containing the labels of each column, st the curve list (either names or the curves themselves), j1 the curve currently treated.

```
r1 = nl_solve('PostEstimate',def);
r1 = nl_solve('PostEstimate',def.Post{1,3});
r1 = nl_solve('PostEstimate',{'disp(1)'});
r1 = nl_solve('PostEstimate',{'Post_NLsolve(1)'});
```

TgtMdlBuild,Assemble

Integrated command to generate linearized models around a specific working point. This command packages the tangent model generation procedures of nl_solve Mode-backTgtMdl. The low level implementations are documented in nl_spring NLJacobianUpdate (for example keepLin interaction are documented there).

- TgtMdlAssemble command outputs a fully linearized assembled model, based on the static state provided.
- TgtMdlBuild command generates a linearized model with superelement coupling containing the tangent stiffness and damping contributions of all non-linearities. The following command options are supported
  - -keepName allows naming the superelements with the non-linearity name.
  - -evalFNL forces recomputation of non-linearities states before generation.
  - -staticInterp generates a tangent model allowing tangent matrix interpolation between different static states. The procedure requires the definition of parameters and a method to compute static states. Static states for MinMax configurations of each parameters is then performed. Matrices showing differences as function of parameters are kept and an interpolation rule is defined using the linear finite element functions of a 2^{npar} vertices hypercube. The output model has stack fields curve,q0 the series of static states with q0.data providing the parameter points, and info,sCoef providing interpolation rules for each matrix.

    ```
    RA=struct('par',Ra,'q0cbk',{{@my_fun,'ComputeStatic'}});
    mo1=nl_solve('TgtMdlBuild-staticInterp',model,RA);
    ```

    Ra is either a Range structure or the content of Range.param (see sdtweb fe_range), q0cbk is a callback in cell-array format.

```
% Linearized model generation
% sample model with cubes in contact
model=d_contact('cubes cbuild');
% resolve static state
q0=nl_solve('static',model);
% linearized model
mo1=nl_solve('TgtMdlBuild',stack_set(model,'curve','q0',q0));
% check the result
feutil('info',mo1)
SE=stack_get(mo1,'SE'); SE{1,3}
```

nl_mesh

Purpose

Integrated mesh modifications and case handling for non-linear applications

Description

Integrated case handling for constraint penalization and coupling component splitting hare implemented in this function.
Some non-linearities require surface/volume remeshing (*e.g.* definition of conforming interfaces for contact) or adaptations (*generation of thin interface layers*). This function regroups such functionalities. Mesh generation are performed by `fe_gmsh`(interface to gmsh) and `fe_tetgen` (interface to tetgen, see `help fe_tetgen`).

`Conform`

The `Conform` call is an integrated call to generate conforming meshes between two facing interfaces. The command generates a conforming surface mesh of the face to replace, merges it with the conform mesh of the second interface, replaces the model face mesh and remeshes the model volume to yield a new equivalent volume with a conform face mesh.

```
mo1=nl_mesh('conform eltsel"FindElt"',model,sel);
% sel={eltSelToReplace eltSelForReplacement;...}
```

`model` is a standard SDT model. `sel` is a cell array containing in each line two `FindElt` commands specifying the element selection face to remesh and the element selection face to use for the conforming interface for replacement.

- Command option `eltsel` allows specifying in a string a `FindElt` command restraining the working area in the original model.
- Command option `smartSize` allows generating a conforming mesh with a coherent mesh characteristic length.
- Command option `gmsh` allows using gmsh to mesh the final volume.
- Command option `tetgen` allows using tetgen to mesh the final volume (by default).
- Command option `output` asks to output the generated mesh in a `.mat` file.
- Command option `OrigContour` asks to keep original positions of mid-nodes of the quadratic faces delimiting the volume to remesh. This may however yield mesh wrapping problems when the face to remesh is much coarser than the mesh trace to place for conformity.

- Command option `mergeTo` allows specifying a `FindElt` selection command in a string to replace the mesh on another model selection than the one used to generate the conforming interface (which uses `eltsel`.
- It is also possible to provide additional arguments, which will be passed the the `nl_meshcover` call performed in the procedure.

Limitations: The `Conform` call only supports generation of conforming interfaces when one interface contour fully contains the other interface contour. Handling of more complex contour configurations has not been implemented. Besides, this function has been designed to handle planar surfaces. Additional operation to work on non planar surfaces are left to the user (*e.g.* pre/post projections of the surfaces on a plane).

`Contour`

Call `ContourFrom` generates SDT `beam1`or `beam3` contour models for CAD definitions. All formats readable by `gmsh` can theoretically be used. Only the `.geo`, `.stp` and `.igs` are tested.
Since `.geo` files can contain geometric yet non discretized objects, a 1D meshing pass is performed with `gmsh` to provide an SDT contour model. This is not supported for other file types.

```
model=nl_mesh('contourFrom','file.stp'); % not specifying the type
```

Call `Contour` generates an SDT face mesh from an SDT `beam1`or `beam3` contour.

```
model=nl_mesh('contour',model);
```

`model` is an SDT beam model defining a closed contour.

- Command option `lc`*`val`* allows specifying a characteristic length for Gmsh.
- Command option `lcmin`*`val`* allows specifying a minimal characteristic length for Gmsh.
- Command option `quad` allows generating quadratic meshes.
- Command option `keepNode` asks to keep the original contour `NodeId` for the `contour` command.
- Command option `diag` asks to output the Gmsh log file for diagnostic problems.
- Command option `single` tells `nl_mesh` that a single contour is defined. This is useful when several closed contours are defined since it is impossible to automatically decide whether each contour is independent or if they define a single complex contour.

- Command option `group`*val* is used in combination to the `s`ingle command option. This allows specifying which contour group will be meshed, while other possible contours will define holes.

- Command option `algo`*''val''* allows specifying which algorithm `gmsh` must use (this depends on the `gmsh` version, report to the `gmsh` documentation for more details).

- Command option `AllowContourMod` allows `gmsh` adding nodes on the contour provided. By default `gmsh` is forced not to add nodes to the lines defining the contour to mesh.

Cover

The `Cover` call is designed to mesh the interstice between two closed planar contours, when one fully contains the other. The call is performed as

```
[newModel,opt,largeContour]=nl_mesh('cover',model,{eltsel_large,eltsel_small});
```

`model` is a standard SDT model. Variables `eltsel_large` and `eltsel_small` are `FindElt` calls defining the element selection of the respectively large surface and small surface (the small being contained in the large).
The output `newModel` is the mesh generated from the surface contours.
`opt` outputs additional information about the mesh generation, it is a `struct` containing fields `.NodeAdd` specifying the potential nodes added in the interstice space meshed,`.nodeEdgeSel1` specifying the `NodeId` of the nodes located on the `eltsel_large` contour, `.nodeEdgeSel2` specifying the `NodeId` of the nodes located on the `eltsel_small` contour, and `.tname` the name of the temporary file containing the generated mesh.
`largeContour` provides the original contour in beam elements of the `eltsel_large` selection.
The following command options are available

- `merge` allows merging the interstice mesh with the inner mesh of the `eltsel_small` selection.

- `quad` allows generating proper quadratic meshes.

- `smartSize` allows generating an interstice mesh with a characteristic length in coherence with the contour mesh length.

- `lc`*val* allows setting the characteristic length to *Val* to the interstice mesher.

- `algo`*''name''* allows specifying the meshing algorithm *name* to the `gmsh` mesher. See the `gmsh` documentation for more information.

`Hole[,Groups,Diff,Drill,Gen]`

The `Hole` command series aims at handling hole detection on surfaces and bore drilling generation. The following functionalities are avaiable

Command `HoleGroups` detects holes on a closed surface and outputs a contour model with element groups relative to each isolated contour. A second output provides the `GroupId` corresponding to detected holes.

Command `HoleDiff` provides surface elements that are inside the holes of a given contour. You should better exploit `lsutil`to get a robust result.

Command `HoleGen` generates a planar surface with a ruled mesh featuring a hole and controlled radial positions.

- `.len` length of plate
- `.wid` width of plate
- `.rAnulus` radii for base positions
- `.ND` angular refinement
- `.NRext` external to bolt radius refinement
- `.MatId` assign mat/pro id to meshed part
- `.noExt` remove exterior side
- `.Center`
- `.normal`

Command `HoleDrill` generates cylindrical drills in a model with the possibility to integrate a ruled bolt mesh.

`Replace`

The call `Replace` is designed to replace parts of a model mesh with new given meshes, mesh parts conformity is assumed. It is performed as

```
model=nl_mesh('replace',model,nodesToReplace,NewModel,nodeIDtoKeep)
```

`model` is a standard SDT model. `nodesToReplace` is a cell array containing vectors of `NodeId` specifying the areas to be replaced. `NewModel` is a cell array containing the new models which will be merged to the mesh in coherence with the removed elements (specified by `nodesToReplace`). `nodeIDtoKeep` is an optional argument specifying `NodeId` of the original model for nodes whose `NodeId` must not change in the transformation.

Control of `nodeIDtoKeep` per `NewModel` part is possible by providing a cell array of `NodeId` list of the same size than `NewModel`.
The following command options are available

- `setMat` allows defining a specific `MatId` to the output mesh.
- `setPro` allows defining a specific `ProId` to the output mesh.
- `eltset`*`FindEltString`* can be provided to provide an element selection for `MatId` and `ProId` assignment.
- `keepNoCheck` in combination with the use of a third argument `nodeIDtoKeep` assumes the nodes numbering is correct and forces the nodes original numbering without check.
- `-jAll` asks to join all elements per type, then separated by `MatId`
- `-inSet` asks to maintain coherence with `EltId` sets. `EltId` sets for which the totality of a given removed part belonged to will be updated to contain the `EltId` of the replacement mesh.

`Rivet`

This command generates rivet drills in a specified contour. A model containing a beam contour can be provided, or an `EltSel` string generating a surface selection (see section and the `selface` option) on a bigger model. A data structure providing the origins, and rivet radius and washer (or rivet head radius). The mesh generated between both radius is structured.
The data structure must contain fields

- `Orig` providing the rivet centers in an `[x y z;...]` matrix.
- `radHole` providing the rivet hole radius, either a scalar if all rivets have the same radius, or a line vector providing each rivet radius separately.
- `radWash` providing the rivet washer (or head) radius, either a scalar if all rivets have the same washer radius, or a line vector providing each rivet washer radius separately.

and can optionally contain fields

- `plane` To directly provide the contour plane normal to define the drilling, in an `[nx ny nz; ...]` matrix.
- `Ns` To define the number of mesh segments in the rivet to washer radius area (default 10), either a scalar if all rivet heads have the same properties, or a line vector defining the property for each rivet separately.

- Nr To define the number of mesh radial nodes in the rivet to washer radius area (default 2), either a scalar if all rivet heads have the same properties, or a line vector defining the property for each rivet separately.

- Command option MatId*v*al allows setting the modified mesh MatId to *v*al.

- Command option ProId*v*al allows setting the modified mesh ProId to *v*al.

- Command option -fill outputs in second argument a compatible mesh of the rivet bores.

- Command option -allQuad outputs the remeshed model with elements only.

Following example meshes a rectangular contour with a few rivet drilling inside.

```
% Generate a global contour
model=struct('Node',[...
 1 0 0 0  0  0 0;
 2 0 0 0 10  0 0;
 3 0 0 0 10  2 0
 4 0 0 0  0  2 0], 'Elt',[]);
model.Elt=feutil('ObjectBeamLine 1 2 0 2 3 0 3 4 0 4 1',model);
model=feutil('refinebeam .2',model);
%feplot(model)

% define rivet positions, eventually planes
RO=struct('Orig',[ 3 1 0;6 1 0;9 1 0],...
 'radHole',[.2;.2;.2],...
 'radWash',[.8;.8;.8]);

model=nl_mesh('Rivet',model,RO);
cf=feplot(model);
```

GmshVol

This call integrates the generation of a volume mesh from a face mesh with gmsh.

```
model=nl_mesh('GMSHvol',model);
```

`model` is a standard SDT face mesh model.

- Command option setmat allows specifying a specific MatId to the output mesh.

- Command option `setpro` allows specifying a specific `ProId` to the output mesh.
- Command option `keepFaces` asks to keep original `NodeId` of the nodes located on the face mesh.
- Command option `lc` specifies a characteristic length for gmsh.
- Command option `clmin` specifies a minimal mesh length for gmsh.
- Command option `clmax` specifies a maximal mesh length for gmsh.

`ExtrudeLayer`

This command generates a non trivial extrusion of a face mesh following the face normal at each node, to generate a volume layer.

```
model=nl_mesh('ExtrudeLayer thick Val',model);
```

`model` is an SDT model with shell elements (a surface definition).
Command option `thick` specifies the extrusion thickness. Command option `setmat` allows specifying a specific `MatId` to the output. Command option `setpro` allows specifying a specific `ProId` to the output.

`StackClean`

This call cleans up a model stack when mesh modifications have been performed. It cleans up stack entries definition that became incoherent with some mesh modifications.

```
model=nl_mesh('StackClean',model);
```

Command option `rmuns` removes stack entries that could not be sorted out. Command option `rmmod` removes stack entries affected by the model modifications.

See also `celas`, `p_spring`, `fe_gmsh`

spfmex_utils

Purpose

OfactOptim

This command can be used to set spfmex parameters in order to optimize computation speed for factorization and / or solving.
`spfmex_utils('OfactOptim',ki,RO,ofact(1,'lu'));`

`ki` is the matrix that is used for the optimization. `RO` is a data structure defining options with following fields:

- `.nCompt` Number of computation for result averaging.
- `.maxDomain` Max size of blocks of the elimination tree (fraction of matrix size).
- `.maxZeros` Max number of zeros in the blocks of the resolution tree (fraction of matrix size).
- `.refineStep` Number of step to refine the optimal parameter pair found in the first step. Command option `-refine` must be added to perform the refine step.

The last argument `ofact(1,'lu')` is needed in order to call directly `spfmex_utils`.
Available command options are

- `-setopt` use default `RO`.
- `-refine` performs refine step for optimal search.
- `fact` to benchmark factorization step.
- `solve` to benchmark resolution step.
- `-plot` to plot history in iiplot

Following example optimize only solving:

```
ki=rand(20);
RO=struct('nCompt',100,... number of computation for result averaging
          'maxDomain',2.^[4:7],... parameter 1
          'maxZeros',logspace(-3,1,5),... parameter 2
          'refineStep',3); % refine results to most relevant parameters
spfmex_utils('ofactoptim solve-refine',ki,RO,ofact(1,'lu')); % method,solve,fact,-setop
```

nl_bset

Purpose

Non linearity to support handling of enforced displacement

This is implemented in `nl_spring('nl_bset')`. This currently assumes the existence of a stiffness and xxx viscous damping xxx.

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