Orthogonal Maximum Sequence Sensor Placements Algorithms for modal tests, expansion and visibility.

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ABSTRACT

After discussing standard requirements on sensor configuration and addressing the issue of non triaxial measurements, the paper introduces a new class of sensor placement algorithms that locate sensors at the maximum response position of an orthogonal sequence of vectors. The methodology is first applied to target modes and provides similar results to standard placement algorithms at a fraction of the numerical cost. *Fixed sensor modes* are then defined and shown to provide an efficient mechanism to validate a given placement and place additional sensors. A placement strategy to detect modeshape changes is finally proposed to enhance the visibility of selected model parameters.

1 INTRODUCTION

When placing sensors for a modal test, one can distinguish two main objectives [1]. Some sensors are placed to allow a direct validation of measurement quality during testing. This implies that a sufficient number of sensors are placed in a regular fashion to allow the generation of a test wire-frame that animates properly. Some levels of redundancy and constraints on actually accessible positions place other demands on the placement. Once test validation taken care of, other sensors can be placed with correlation objectives.

Placement is a combinatorial problem where one seeks to place N_S sensors out of N_{pot} potential locations. The test of all possibilities is rarely realistic, one can thus consider sets of N_S sensors simultaneously (simulated annealing, genetic searches), in substitution algorithms, or sequential approaches where one removes (decimation) or adds (aggregation) one sensor at a time (see Ref. [2] for a discussion of existing algorithms).

The constraints on wire-frame validity and sensor locations gives a strong motivation to use aggregation algorithms gradually building the sensor set as proposed in Ref. [1]. Proposed placement algorithms however tend to retain combinatorial costs and thus be difficult to use when dealing with models containing a few hundred thousand nodes.

The present paper analyses a class of placement algorithms that aggregate sensors sets one sensor at a time at the location of the maximum response of the current vector of an orthogonal sequence of vectors (hence the name OMS for Orthogonal Maximum Sequence). The sequence is orthogonal in the sense that each new vector is built to be zero when observed with the currently placed sensors. Various subspaces used to start the sequence will be considered in the paper.

Section 2 discusses standard difficulties in placing sensors for test validation. Section 3 discusses some of the well known placement algorithms and details the procedure of the new OMS algorithm to place sensors to distinguish target modes.

In section 4, one defines fixed sensor modes which are directly related to *fixed interface modes* used classically in Component Mode Synthesis [3]. These modes give a direct indication of the frequency limit of static expansion and it is shown how they can be used to add a few sensors on a given test configuration.

Finally section 5 introduces the idea of placing sensors to make the test sensitive to specific parameters in the model. A subspace of directions orthogonal in mass to the nominal target modes and in observation with the current sensor set, is generated and can be used to place sensors for visibility objectives.

2 PLACEMENT FOR TEST VALIDATION AND CORRELATION

The first objective of a placement is to allow the test engineer to validate the quality of his test. In particular, this means that correct animation of mode shapes is needed. Such animation requires alignment of sensors and estimates of 3D motion. The second objective is to allow a proper test/analysis correlation which requires a good topology correspondence between test and analysis.

Rather than giving a set of general rules, one will illustrate typical pathologies using the test configuration of a car door shown in figure 1. The pathologies of this configuration are well understood and now dealt with through proper test design procedures (in particular when using optical measurements [4]). The author is grateful to be able to use this non trivial test as an example.

The test geometry is not regular. When looking at the undeformed wire-frame it is difficult to see if the structure is deformed or not. A rule for test placement is that one should whenever possible used structured meshes with regularly spaced points on a surface. A related constraint in building a wire mesh is that the wire-mesh should be decomposed in parts that correspond to parts of the structure. Here, one properly connects sensors on the glass and the door frame in two groups.

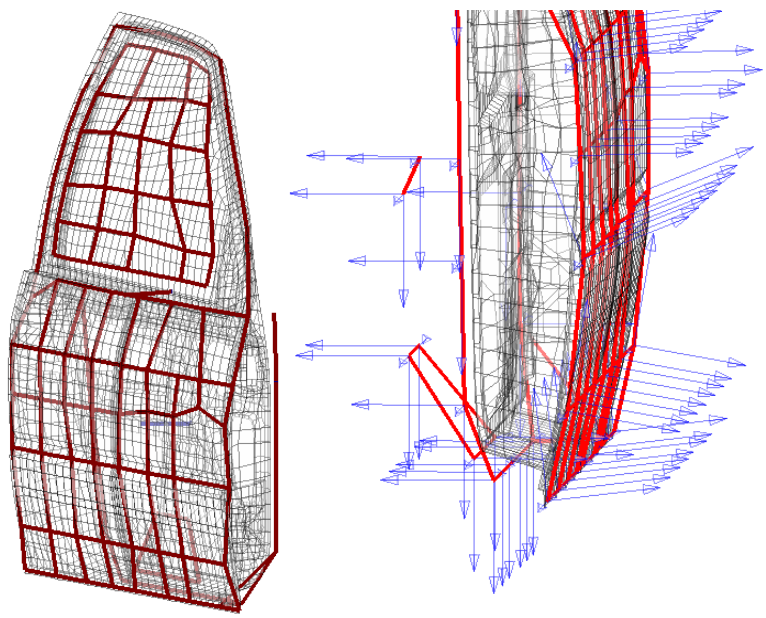

Figure 1: FEM and test meshes of a car door.

A second problem found in this sensor configuration is linked to the use of non triaxial measurements. As clearly shown in the figure, horizontal rows of monoaxial sensors where placed at various elevations on the structure. The angle between the two bottom rows does not correspond to the general curvature of the door, as a result direct animations of the test wire frame seem to indicate in plane motion of these two sensor rows when really the motion is mostly transverse to the surface.

Such pathologies are typically been cited as arguments to use triaxial measurements despite the obvious waste of channel count [1]. One reason often mentioned is that building interpolations by hand can be difficult. The solution implemented in the Structural Dynamics Toolbox [5] is to assume that the wire-frame is a lattice of beams and to use mode shape expansion techniques [6] to interpolate non measured displacements. Since modes of the wire-frame are not representative, static expansion is used. Beam sections are based on the average inter-node distance.

To validate this procedure, one considers the analytical modeshapes of the door structure, computes their observabilities at the sensors and expands the response at each test node. In the original wire-frame, the window is not connected to the frame and only transverse measurements are used. To obtain a realistic in-plane motion of the window, the four additional connections (shown in red in figure 2) are created. The resulting expansions are very close approximations of the actual response. The MAC values comparing observed analytical shapes in three directions and expansion of the restricted sensor set are superior to .9

for the first 50 modes. The two overlaid deformations in figure 2 show that the major source of differences is linked to the row of sensors on the frame at the bottom of the window. These difficulties would be significantly alleviated if this row of test nodes was strictly aligned.

The author's practice is that wire-frame expansion eliminates all of the tedious parts in reconstructing motion in unmeasured directions and rarely generate distortions in the interpretation of test results. The need for triaxial measurements is thus much lower than typically thought.

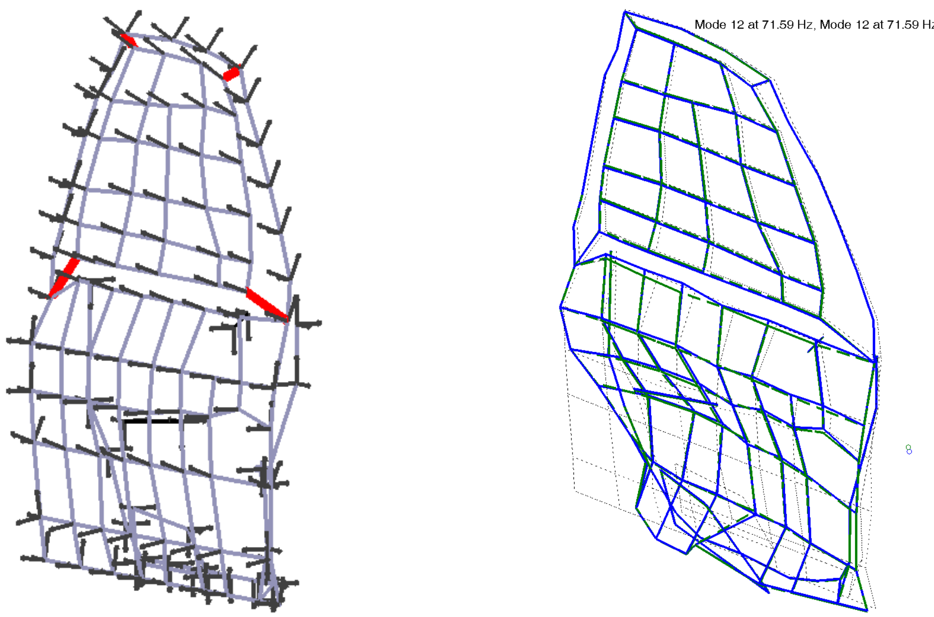

Figure 2: Wire-frame and additional links for wire-frame expansion in red. Illustration of problems in the wire-frame expansion.

The last pathology of this test configuration is the non-coincidence of the test and FEM geometries. A number of test nodes are actually positioned away from the structure (as apparent at the bottom of door in the figure but also true one the window frame). When these errors are due to poor measurements of the sensor locations, an orthogonal projection on the nearest FEM surface can be used (SDT `stick` operator). Accounting for differences between element surface or neutral fiber and sensor position can be important in certain applications [7]. Finally the FEM geometry is often not quite exact. For automotive applications, but more generally for structures made of assembled shells, the response is fairly sensitive to geometry tolerances of the order of the shell thickness, measuring the exact geometry of the test piece is then critical for good test/analysis correlation.

3 PLACEMENT STRATEGIES FOR INDEPENDENT MODES

3.1 Potential sensors

At the beginning of a placement, one starts by defining potential sensors by their observability matrices

$$\{y_{pot}\} = [c_{pot}]\{q\} \tag{1}$$

It is useful to note that the formalism of observation matrices is quite general and allows the simultaneous consideration of translation, rotation or strain sensors.

In most applications, sensors are placed to observe a set of target modes (although alternative objectives will be proposed later). The objective of classical placement algorithms is thus to select a subset of potential sensors that optimizes some characteristic of the modal observability matrix

$$[c_{pot}][\phi_j] \text{ with} j \in \text{target set} \tag{2}$$

Physical placement of sensors is easier, if they are attached directly to the surface. For accelerometers, one creates a list of potential sensors by placing a first direction normal to the surface and then defining two orthogonal directions. Defining a preferred plane (horizontal for example) for the second sensor is also useful. Such definition of potential accelerometer locations can be automated for shell, volume and beam elements. Connections between different element types lead to difficulties in defining directions but the associated nodes can be eliminated in most FEM models without removing any significant potential area.

In a second step, locations associated with parts that are difficult to access (the sensor cannot be placed there in practice) and areas that have local modes in the bandwidth of interest (unless one really seeks to characterize these local modes) should be eliminated. Doing an *a priori* elimination of these locations is tedious if not impossible. It is thus much more practical to place sensors gradually while allowing the user to relocate proposed sensors to close but more appropriate locations from a test engineer point of view.

3.2 Classical strategies

The Effective Independence algorithm [8, 9] (and its variants [10, 11]) seeks to maximize the linear independence of target modes observed at sensors. This is obtained by maximizing the determinant (or an approximation of the determinant) of the Fisher information matrix

$$[Q] = [[c_{pot}][\phi_{1:NM}]]^T [[c_{pot}][\phi_{1:NM}]] \tag{3}$$

Another classical placement objective seeks to generate modal observation matrices that are nearly orthogonal. In Ref. [1], it is thus proposed to minimize the off-diagonal terms in the MAC matrix

$$\mathsf{MAC}_{jk} = \frac{\left(\{\phi_j\}^T [c]^T [c] \{\phi_k\}\right)^2}{\left(\{\phi_j\}^T [c]^T [c] \{\phi_j\}\right)\left(\{\phi_k\}^T [c]^T [c] \{\phi_k\}\right)} \tag{4}$$

A related objective might be to be able to minimize the MMIF [12] using inputs collocated with the sensors. One would thus compute the MMIF at resonance

$$q(\omega_k) = \min_{\{u\}\neq 0} \frac{\{u\}^T \mathsf{Im}[H]^T \mathsf{Im}[H] \{u\}}{\{u\}^H [H]^H [H] \{u\}} \tag{5}$$

with

$$[H(\omega_k)] = \sum_j \frac{\left[c\phi_j \phi_j^T c\right]}{\omega_j^2 - \omega_k^2} \tag{6}$$

and minimize the mean of $q(\omega_k)$ over the target modes.

While these objectives have been shown to be efficient, they have some shortcomings. They fail to take into account the spatial relation between potential sensors. In the resulting placements, one often finds very close sensors as shown in figure 3. They use fundamentally combinatorial approaches so that the placement becomes a slow procedure for large models and is thus not well suited for interaction with the engineer performing the placement.

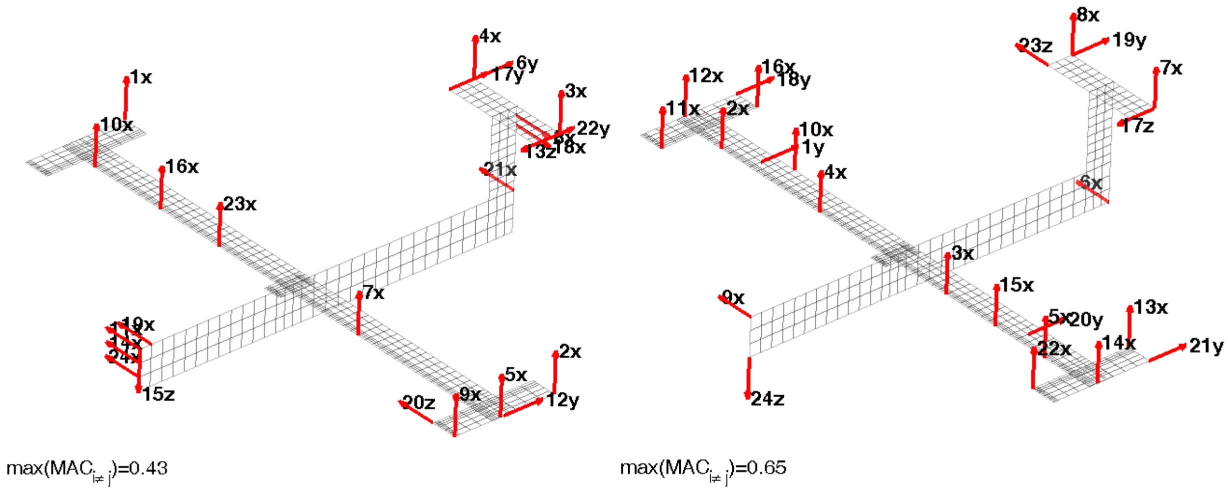

Figure 3: Sensor placement using 24 target modes. Min off-diagonal terms (left), EfI (right).(GARTEUR SM-AG-19 Testbed model [13])

3.3 OMS placement for target modes

This study proposes a new class of placement algorithms called OMS, for Orthogonal Maximum Sequence. The initial argument for these algorithms is that sensors should be placed where the response is maximal. The response is assumed to be characterized by a set of vectors of equal importance stored in a matrix T^0. For mode independence, the subspace contains the collection of target modeshapes $T^0 = \left[\phi_{\text{target}}\right]$. Other subspaces will be considered in the following sections.

If the vectors are ordered, one can consider the maximum response at the current vector T_k^k. For modeshapes, one thus places sensor k to distinguish mode k from lower modes. An alternative defines the maximal response (associated with sensor k) as the sum of the absolute values of modal observabilities at potential sensors

$$k = \arg\max_{m \in pot} \sum_j |c_m\{T_j^0\}| \tag{7}$$

Since there is only one location k where the response is maximal, one needs a mechanism to place the next sensor (that is define a series of subspaces T^k). The proposed approach is to use a recursive orthogonalization of T^k with respect to its observation by the currently placed sensors. To place sensor $k+1$, one thus seeks to build a subspace T^k such that $c_m T^k = 0$ for $m < k$. This is easily obtained using the following recursion

$$\left[T^{k+1}\right] = \left[T^k\right] - \left[T^k\right]\left\{c_{k+1}T^k\right\}^T \left(\left\{c_{k+1}T^k\right\}\left\{c_{k+1}T^k\right\}^T\right)^{-1} \tag{8}$$

While this methodology is inherently associated with an aggregation strategy (one adds sensors one by one), it presents a very significant speed advantage. Since the placement is based on a simple maximum search, the placement is no longer a combinatorial problem and there is no need to restrict the number of potential sensors and algorithm can be used on very large models.

Figure 4 illustrates placement results for a model of the Ariane 5 ESC-A [14]. On the left, the first 10 sensors are placed in a very logical order. The first two modes, are associated with bending of the upper payload (hence two orthogonal sensors on the top). Sensors 3,4 are for the bottom payload. The basis for the following sensors is also simply related to the next modes.

The right plot shows an interesting trend where a ring of sensors is placed around the oxygen tank. This is a case where there are a number of ovalization modes of this tank. If these modes are considered important, the ring of sensors is indeed needed to distinguish the modes. If they are considered as local modes of lesser interest, then one could simply eliminate the tank surface from the potential sensor locations.

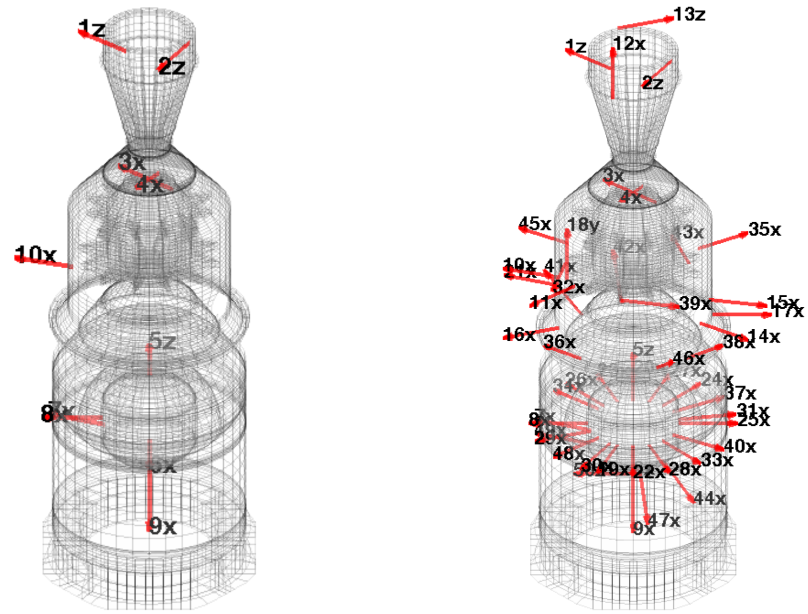

Figure 4: Sensor placement for 10 target modes. First ten sensors (left). 50 sensor placement (right).

One difficulty with sensor placements generated from scratch, is that they tend to be rather irregular. This is not acceptable when trying to generate a usable wire frame display of the test configuration. A reasonable placement strategy thus work as follows: place a few sensors, correct their positions, possibly add a few for the sake of wire-frame displays. Then figure out where to add the next few. The OMS algorithm can easily be used in conjunction with restarts. One just generates the orthogonal sequence for current sensors before placing a new one. The OMS algorithm can however only place as many sensors as shapes in the subspace. Adding more target modes in the subspace is not particularly interesting so that the new notion of fixed sensor modes will be presented in the next section.

4 OMS PLACEMENT FOR EXPANSION

Experimental measurements are incomplete : even with ESPI there are hidden faces and with 3D laser vibrometry the number of measured points is limited. It is thus important to estimate motion elsewhere than sensors. The general process, called expansion in the modal analysis literature, is to use a model of how the structure behaves to estimate responses at other locations or in other directions. A review of expansion methods is made in Ref. [6].

The ability to expand is typically linked to the ability to distinguish vectors of a particular subspace in which one seeks the expanded responses as a linear combination of base vectors. This is easily understood for the subspace of rigid body modes. Six translation sensors at a least 3 locations are needed to distinguish all six rigid body modes.

The next question is obviously the definition of *appropriate* subspaces. Static and modal expansion are the two most common expansion methods. The subspace of target modeshapes was considered in the previous section and one can analyze the derived placement algorithm as a placement for modal expansion.

To derive a placement for static expansion, one needs to introduce the important subspace of fixed sensor modes. As shown in [6], static expansion tends to become inaccurate when the frequency is high enough for inertia effects of nodes between sensors to become important. One can thus solve the eigenvalue problem

$$\left[K - \omega_j^2 M\right]\{\phi_j\} = \{0\} \text{ with } [c_m]_{m \leq k}\{\phi_j\} = 0 \tag{9}$$

where the response at sensors is constrained to be zero. These shapes are a generalization of fixed interface modes in Component Mode Synthesis [3] and their frequencies give a direct indication of the frequency limit of accuracy of static condensation [15]. Numerically, they are computed by generating a basis for the null space of $[c_{1:k}]$ and computing modes within this subspace.

Figure 5 shows the first fixed sensor mode associated with initial sensor configuration of the car door. This mode is a first bending of the internal door pannel. This is quite coherent with the fact that only the external side was instrumented. The second information of interest is the frequency of that mode 76 Hz. This is a fairly low frequency (only six modes of the free structure are below that frequency). Static expansion would thus only be valid for very few modes.

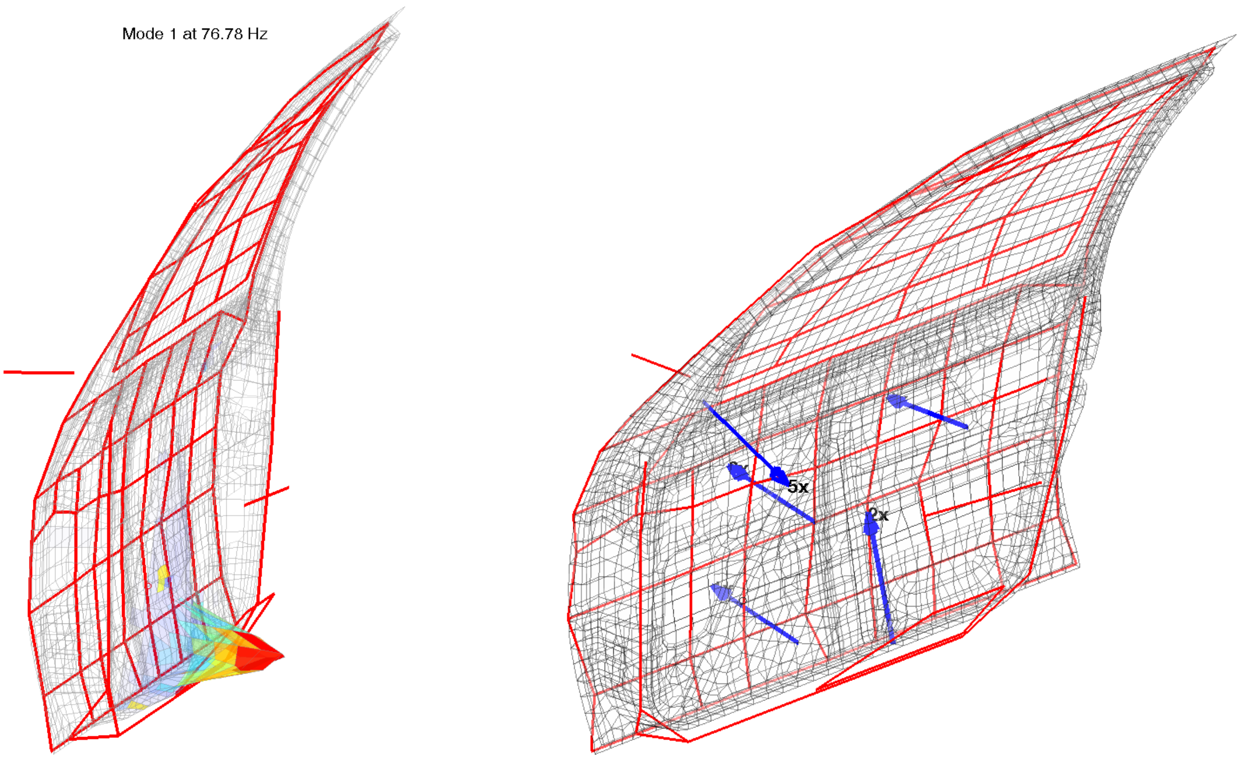

Figure 5: Left : First fixed sensor mode of car door model. Right : 5 sensors placed using the OMS placement on fixed sensor modes.

The OMS algorithm for static expansion can place an additional sensor at the maximum response of the first fixed sensor mode and recompute new fixed sensor modes with the new placement. This approach tends however to be computationally intensive, so that it can be preferable to place a few sensors at a time using the same number of fixed interface modes.

The left plot in figure 5, shows 5 sensors placed using the OMS placement on fixed sensor modes. Two are placed on to stiffen the interior panel modes. Two are placed near the attachments of an internal stiffener beam that has relatively low frequency modes. One is placed near the mirror attachment which exhibits a fixed sensor mode a 98 Hz.

With these five additional sensors, the first fixed sensor mode is again a bending of the internal stiffener beam. This indication gives a strong incentive to place sensors directly on this beam even though this is technically more difficult (this corresponds to a change in the list of potential sensor locations).

A classical criterion to measure the improvement in the configuration is to compute the mass cross orthogonality between original modes ϕ_j and static expansion of their measurements. As shown in figure 6 the added sensors improve all modes and some significantly. The POC is still not perfect but this is coherent with the fact that one still has a fairly low frequency fixed sensor mode.

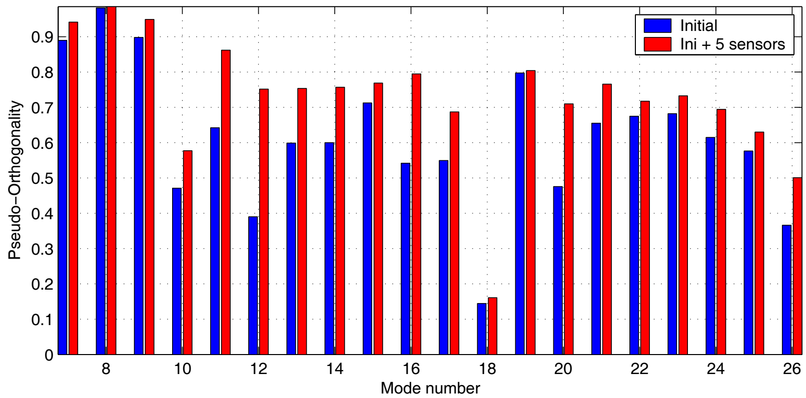

Figure 6: Pseudo orthogonality check evolution with 5 new sensors.

5 OMS PLACEMENT FOR PARAMETER VALIDATION

Gradual placement of sensors using fixed sensor modes is a very efficient and practical tool. It does not however answer an important question for model validation : where should I place sensors to be sensitive to a given model parameter.

The heuristic principle for the proposed approach is that a sensors for a visibility objective, sensors should be placed in a way that will allow the detection of changes in the shape of target modes. Changes in target modes can be characterized simply by mode shape sensitivities (for their computation see Ref.[16] for example). But what one really wants is to detect modeshape changes. One will thus consider the components of the target modeshape sensitivities that are mass orthogonal to the initial modes

$$T^A = \left[\frac{\partial \phi_{targ}}{\partial p}\right] - [\phi_{targ}]\left[[\phi_{targ}]^T[M]\left[\frac{\partial \phi_{targ}}{\partial p}\right]\right] \tag{10}$$

This subspace is not appropriate for an OMS placement because sensors for visibility are placed after sensors to observe mode shapes. The objective is thus to generate shapes that are orthogonal in some sense to the current observation matrix. Assuming that the current setup would be valid for static modeshape expansion, one can define *constraint modes associated with sensors* [3] as static responses to unit displacements on sensors with reaction forces R_j on the same sensors. Constraint mode j thus verifies

$$[K]\{T_{j,\text{cons}}\} = \left[c_{1:k}^T\right]\{R_j\} \text{ with } [c_{1:k}]\{q_j\} = \delta_{jk} \tag{11}$$

Given these shapes, one can start and OMS placement using a subspace where one removed static contributions of sensors from subspace (10)

$$\left[T^0\right] = \left[T^A\right] - \sum_{j=1}^{k}\{T_{j,\text{cons}}\}[c_j]\left[T^A\right] \tag{12}$$

Figure 7 shows two shapes associated with subspace $\left[T^0\right]$ for a change in the window/frame joint. The indications of frequency are found by solving the reduced eigenvalue problem

$$\left[T^0\right]^T\left[K - \omega_j^2 M\right]\left[T^0\right]\{\phi_{jR}\} = \{0\} \tag{13}$$

Since the frequencies are low, one can expect shape changes to be relatively significant. The shapes are also quite reasonable. A change in the joint stiffness affects the behavior of the mirror attachment area and the bending/torsion coupling of top part of the interior panel. Both type of motion are not observed in the nominal test configuration which focuses on the external frame.

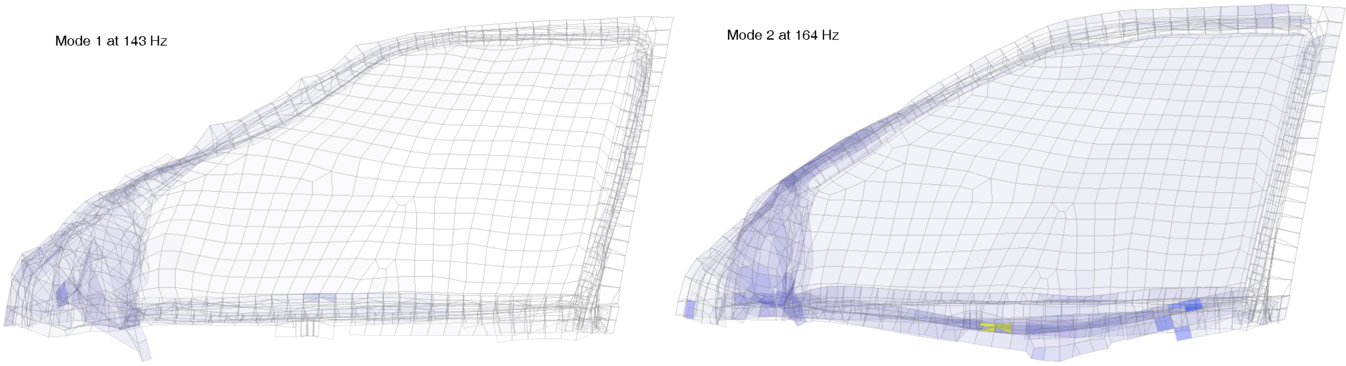

Figure 7: Orthogonal sensitivity directions associated with the window/frame joint.

An illustration of the validity of this placement procedure for applications in error localization with energy criteria can be found in Ref. [17].

6 CONCLUSION

The proposed class of OMS placement algorithms gives very good results, that are most of the time very coherent with the test engineer's intuition. The numerical cost of placing new sensors in marginal even for very large models.

The concept of fixed sensor modes provides a new mechanism to validate and enrich (through an OMS procedure) a given sensor configuration. Implementation of fixed sensor mode computations is immediate in SDT [5] and not difficult in NASTRAN (through the use of Multiple Point Constraint associated with observation equations).

The detection of shapes associated with the part of modeshape sensitivities that is orthogonal to target modes provides a mechanism to enhance the visibility of changes to specific parameters in a model.

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