Using Expansion and Interface Reduction to Enhance Structural Modification Methods

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ABSTRACT

Structural modification techniques have the general objective of predicting the effect of a modification on the dynamic behavior of a structure that is only known through test results. While the validity of the method is established for cases were the modification is connected to the base structure at discrete points, obtaining good predictions for distributed modifications is still considered a major problem. The paper extends the classical theory of structural modification by coupling it with modeshape expansion and Component Mode Synthesis with interface model reduction. While combining these well-established methods, specific care is taken to give the user criteria on the validity of the predictions. The proposed methodology is applied to the case of a stator casing in an electricity production plant. The method is first validated on a finite element model that is representative of the real structure. Predictions based on the real test are then obtained and compared with the test performed after modification.

1 INTRODUCTION

Structural modification methods use experimental models to obtain predictions of the response of structures after modification. While modal and FRF based structural modification methods have been used for years (see [1] for a review), very few studies address the case of distributed modifications [2, 3] or allow for test setups designed with no a priori knowledge of the modification geometry.

The present study revisits modal based structural modification methods by coupling them to test shape expansion and Component Mode Synthesis with interface model reduction.

Shape expansion [4] is used to estimate the responses of all degrees of freedom of the interface between the base structure and the modification. Doing this implies the use of a partial model of the base structure. This is considered acceptable as long as the partial model is extremely simplified and can thus be built during the testing process. Expansion is then really used as an advanced interpolation method. The use of two different interpolations using or not information about the modification is however used as an indicator for the validity of coupled predictions.

Interface model reduction [5, 6] is used to allow the representation of loads applied on the base structure by the modification through a reduced number of generalized loads that correspond to test sensor locations (the test model is assumed to verify reciprocity so that FRFs to/from all sensors can be predicted).

Section 2 gives theoretical developments needed for the proposed method. Section 3 uses a representative FEM model to simulate test results and allow analyses of the advantages and limitations of the method. Section 4 shows results for the real tests [7]. Sections 3-4 share the same test configuration and same modification model.

2 THEORETICAL BACKGROUND

As shown in figure1, structural modification is well posed as a closed loop prediction problem. The base structure deforms under external loads applied by the modification u_I. With no external loads applied to the modification, loads u_I only depend on interface deformations y_I. The knowledge of the relation between y_I and u_I, that is the existence of a model of the modification, allows a closed loop prediction of the response.

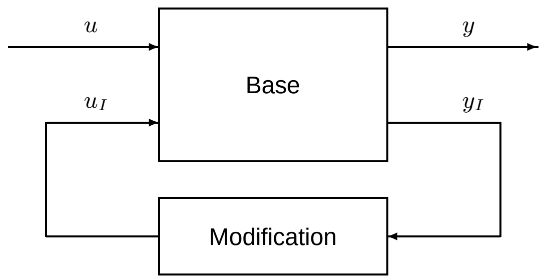

Figure 1: Structural modification as a closed loop prediction problem

2.1 FRF based, multiplicative formulation

For a base structure described by FRFs

$$\{y_I\} = [H_B(s)]\{u_I\}, \qquad (1)$$

and a known dynamics stiffness of the modification

$$\{u_{IM}\} = [Z_M(s)]\{y_{IM}\} \tag{2}$$

one can use displacement continuity $y_{IM} = y_{IB}$ and dynamic load equilibrium $u_{IM} = u_{IB}$ conditions, to obtain the classical expression [1]

$$\{y_{IB}\} = [I + H_B(s)Z_M(s)]^{-1}[H_B(s)]\left\{u_{IB}^{\text{ext}}\right\} \tag{3}$$

The main problem with this approach is the need to invert $[I + H_B(s)Z_M(s)]$. Since near resonances H_B is dominated by the contribution of a single mode, it is poorly conditioned. It is thus often necessary to use regularization techniques (see [8] for example).

2.2 Model based additive formulation

A second class of formulations assumes the existence of a model in the form of a linear differential equation. Here, we will only consider second order models of the form

$$\begin{array}{c} [Z_B(s)]_{N\times N}\{q_B(s)\}_N = [b_{IB}]_{N\times NA}\{u_{IB}(s)\}_{NA} \\ \{y_{IB}(s)\}_{NS} = [c_{IB}]_{NS\times N}\{q_B(s)\} \end{array} \tag{4}$$

where the dynamic stiffness Z is the sum of mass, damping, and stiffness contributions $Z(s) = Ms^2 + Cs + K$, but the same problem could be solved with more general state-space models [9].

Note that the input/output shape matrix formalism used here decouples the choice of DOF q_B from the choice of inputs $\{u_{IB}(s)\}$ and outputs $\{y_{IB}(s)\}$. For full FEM models with compatible interface meshes, b_{IB} and c_{IB} are Boolean matrices. In other words, the u_I form a subset of the load components $F(s) = [b_{IB}]\{u_{IB}(s)\}$. The interest of writing it this way is that transformations of the DOFs are easily translated into transformations of b_{IB} and c_{IB} with no change in the formalism. The test derived modal models considered in the following sections thus keep the same form.

For the coupled prediction, one assumes that the modification is modeled with the finite element method. One can thus always write the modification model in the form

$$\begin{bmatrix} Z_{II}^M(s) & Z_{IC}^M(s) \\ Z_{CI}^M(s) & Z_{CC}^M(s) \end{bmatrix}\left\{\begin{array}{c}\{y_{IM}\}\\ \{q_C\}\end{array}\right\} = \left\{\begin{array}{c}\{u_{IM}\}\\ \{0\}\end{array}\right\} \tag{5}$$

where interface DOFs explicitly appear as DOFs of the model (see [10] for a justification that this is always possible even with incompatible meshes).

Using the general framework of Ritz methods, the coupled prediction is obtained by imposing displacement continuity on the interface ($y_{IB} = y_{IM}$) and projecting the associated model on loads dual to the displacement subspace admissible under the continuity constraint. The projection thus combines continuity and dynamic equilibrium of loads $u_{IB} = u_{IM}$.

For a base model given by (4) and a modification described by (5) this leads to

$$\begin{array}{l}\left[\begin{bmatrix} Z_B & 0 \\ 0 & Z_{CC}^M \end{bmatrix} + \begin{bmatrix} b_{IB} \\ 0 \end{bmatrix}\left[Z_{II}^M\right][c_{IB}\ 0] + \right. \\ \left. + \begin{bmatrix} b_{IB} \\ 0 \end{bmatrix}\left[0\ Z_{IC}^M\right] + \begin{bmatrix} 0 \\ Z_{CI}^M \end{bmatrix}[c_{IB}\ 0]\right]\left\{\begin{array}{c} q_B \\ q_C \end{array}\right\} = F_{ext}\end{array} \tag{6}$$

For q_B et q_C corresponding to FEM DOFs, b_{IB} and c_{IB} are Boolean and (6) corresponds the standard assembly process. For the applications considered here, the q_B are modal coordinates and q_C corresponds to fixed interface modes of a Craig-Bampton model.

2.3 Dealing with continuous interfaces

The assumption, made in the previous section, that all interface DOFs are tested puts a strong limitation on the practical use of the approach. One will thus introduce a distinction between interface displacements $\{y_I\}_{NI\times 1}$, which correspond to the discretization of the modification model (5), and test displacements $\{y_T\}_{NT\times 1}$, which correspond to whatever was acquired during the modal test of the unmodified base. Similarly one will distinguish u_T and u_I.

This highlights the spatial incompatibility problem since in practical applications $NT \ll NI$. To bypass this incompatibility, one will assume that there exists a constant coefficient linear combination relating interface and test displacements for the actual coupled response

$$\{y_I\}_{NI\times 1} \approx [c_{IT}]_{NI\times NT}\{y_T\}_{NT\times 1} \tag{7}$$

This relation imposes a strong constraint on interface kinematics since y_I must be approximated by a subspace of basis T_G whose dimension is smaller than the number of sensors used $NG \leq NT \ll NI$. The choice of this subspace and the justification of its ability to represent the coupled response is a key aspect of the propose methodology.

The construction of a reduced interface model (T_G subspace) is a classical extension of component mode synthesis addressed in references [5, 11, 10, 6] to only cite a few. The results shown in this study use a Craig-Bampton type reduction of the modification where the constraint modes are replaced by the low order modes of the model statically condensed on its interface (as originally proposed in [5]).

For this study, one already assumed that a FEM model existed for the modification. One will further require that a local model of the base exists. The objective of this model is only to allow interpolation of test motion at an arbitrary number of degrees of freedom of the interface between the base model and the modification. This model need not be representative of the base dynamics.

The T_G subspace is here defined on the DOFs of a local model including or not the modification. The extraction of interface motion is thus written as

$$\{y_I\}_{NI\times 1} = [c_{IL}]_{NI\times NL}[T_G]_{NL\times NG}\{y_G\}_{NG\times 1} \tag{8}$$

Given the local model, one can build an observation matrix relating the q_L (DOFs of the local model) with measurements $\{y_T\} = [c_{TL}]\{q_L\}$ (see [12] for possible methods). Thus

$$\{y_T\} = [c_{TG}]_{NT\times NG}\{y_G\} = [c_{TL}T_G]_{NT\times NG}\{y_G\}_{NG\times 1} \quad (9)$$

To estimate generalized motion of the interface, one seeks to establish a relation of the form

$$\{y_G\}_{NG\times 1} = [c_{GB}]_{NG\times NB}\{q_B\}_{NB\times 1} \quad (10)$$

The standard approach, used by subspace based expansion methods [4], solves the least squares problem

$$\{y_G\} = \arg\min_{y_G} \|[c_{TG}]\{y_G\} - [c_{TB}]\{q_B\}\| \quad (11)$$

whose solution is given by

$$[c_{GB}] = \left[c_{TG}^T c_{TG}\right]^{-1}\left[c_{TG}^T c_{TB}\right] \quad (12)$$

which leads to the observation equation (7) with $c_{IT} = c_{IL}T_G c_{GB}$.

Given assumption (8), the second block row of equation (5) describing the motion of the modification becomes

$$[Z_{CC}(s)]\{q_C\} = -\left[Z_{CI}^M(s)\right][T_G]\{y_G\} \quad (13)$$

For the first block row, one further assumes that generalized loads are defined by projection on the subspace T_G. Equation (5) thus becomes

$$\{u_{GM}\} = [c_{IG}]^T\left[Z_{IC}^M\right]\{q_C\} + [c_{IG}]^T\left[Z_{II}^M\right][c_{IG}]\{y_G\} \quad (14)$$

The coupled response is obtained as before by assuming dynamic equilibrium of generalized loads $u_{GB} = u_{GM}$. The model still has the form of equation (6) but one replaces Z_{II}^M by $[c_{IG}]^T\left[Z_{II}^M\right][c_{IG}]$, and Z_{IC} by $[c_{IG}]^T\left[Z_{IC}^M\right]$.

2.4 Error evaluation

Since the predictions made are based on many assumptions, introducing error evaluation tools is essential. The idea implemented in this study is to use two different methods to estimate motion y_I. The first one is given by the coupled response and the observation equation (8). The second is a static expansion $y_{I\text{exp}}$ of the interface response using the local model of the base. This corresponds to an another choice of T_G where the modification is not taken into account.

For a good prediction, one expects $y_{I\text{exp}} \approx [c_{IT}]\{y_T\}$. A visual display can be used to analyze problems but a numerical criterion is needed for interface mode selection. For results shown in the next section, a relative strain energy criterion

$$e_K(y_T) = \frac{\|[c_{IT}]\{y_T\} - \{y_{I\text{exp}}\}\|_{K_M}}{\|[c_{IT}]\{y_T\}\|_{K_M} + \|\{y_{I\text{exp}}\}\|_{K_M}} \quad (15)$$

and a similar kinetic energy criterion e_M were used, with K_M and M_M being the mass and stiffness matrices of the modification condensed on the interface.

3 SIMULATIONS ON A PSEUDO-TEST

3.1 FEM model and test configuration

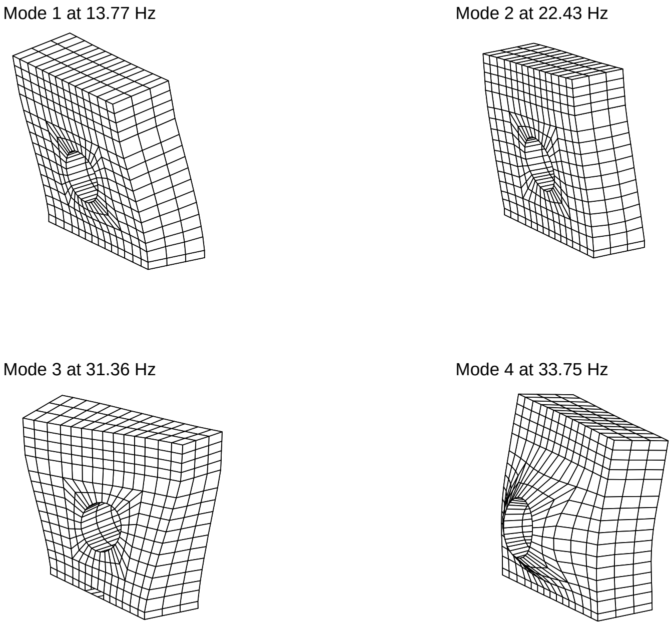

Figure 2: FEM model of base structure.

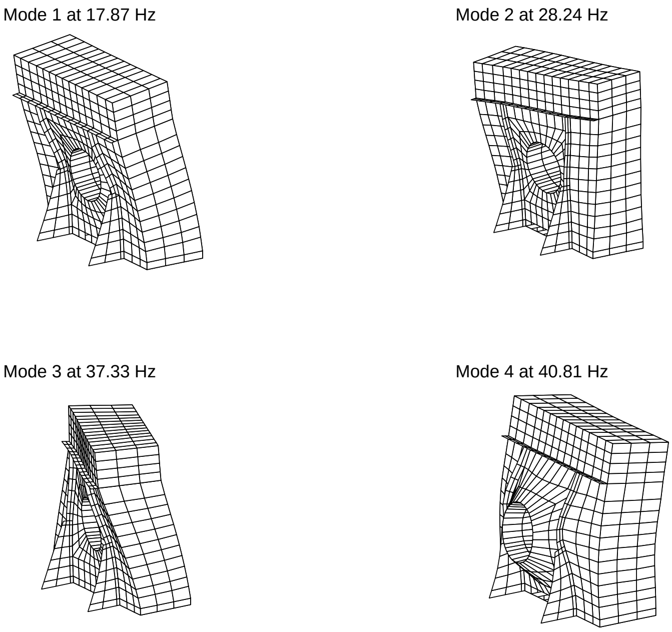

Figure 3: Modified configuration.

To validate the proposed methods, one considers a plate model of a structure similar in shape to the stator casing considered in section 4 and a fairly realistic model of the modification that was really built. The nominal and modified configurations are shown in figures 2-3. All simulations and identification in this study are performed using the *Structural Dynamics Toolbox*[13].

To account for the constraint that the method must not require a FEM model of the base structure, one considers a local model made of the back plate of the casing.

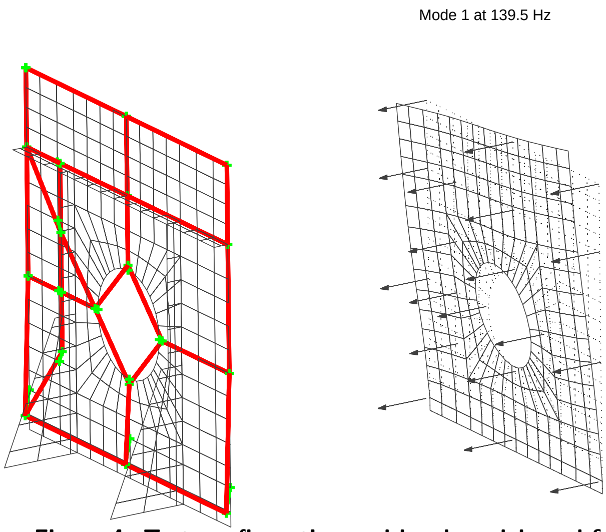

Figure 4: Test configuration and local model used for expansion. First mode with fixed sensors.

A first check on the sensor configuration is to predict the first mode for fixed sensors. If the frequency of this mode is too low, one clearly expect the coupled predictions to be poor since low frequency components of the base response cannot be observed.

The first configuration, shown in figure 4, with only axial sensors (direction y) shows that a bending mode in the xz plane. This motion being important for mode 2 of the base, one can clearly reject the sensor configuration as not acceptable for coupled predictions.

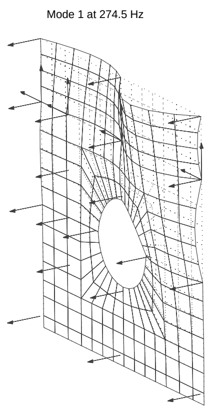

Figure 5: Fixed sensor mode for the retained test configuration.

A second configuration shown in figure 5, keeps 3 additional horizontal measurements at the level of the upper stiffener which leads to a much better behavior with the fixed sensor mode showing high response in an area with few sensors. This final configuration considers 27 sensors.

3.2 Coupled predictions

Figure 6 shows ratios between true and predicted natural frequencies of the modified structure for a varying number of interface modes retained.

For 27 interface modes kept (as many as sensors) predictions are poor. By keeping too many modes, one allows a fairly complex behavior of the interface that is not representative of low frequency behavior.

For 10 interface modes retained, one has good predictions for a base model with and without static correction. While this indicates that obtaining good predictions is possible, it illustrates the need to have indicators allowing this selection.

Note that, the figure is somewhat misleading because automated mode matching, based on the MAC, pairs predicted modes 3-4 to the same true mode 3. But this difficulty is hard to avoid in an automated procedure.

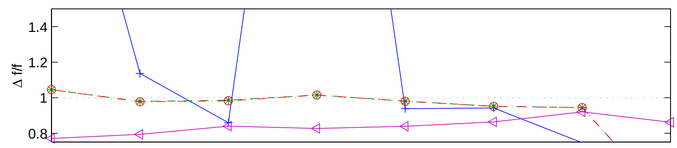

Figure 6: Frequency ratio (predicted/true coupled) with automated mode pairing based on MAC.

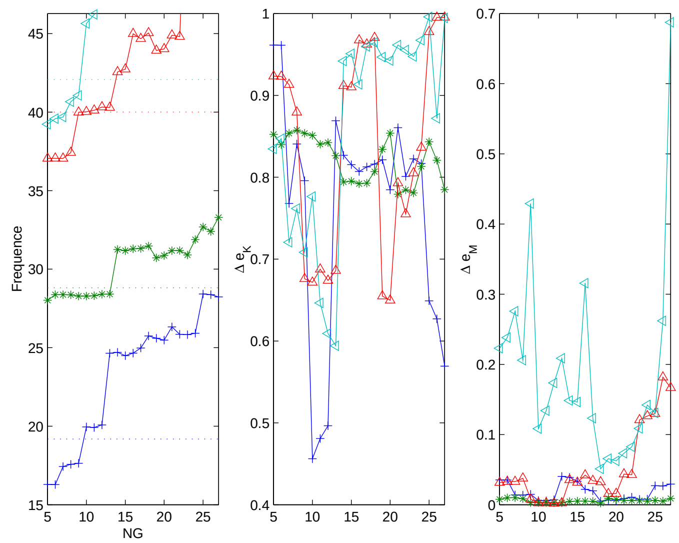

Figure 7: Quality of predictions for the first 4 modes as a function of the number of retained interface modes.

To seek the optimal number of interface modes, one computes the coupled response for each possible value and shows, in figure 7, the evolution of frequencies and energy errors. The strain energy criterion has a relatively clear minimum for 10 interface modes retained. The proposed selection strategy thus seems applicable.

3.3 Transfer functions and sensitivity

The previous section focused on modal frequencies. One now seeks to analyze performance for FRF predictions. One considers the first input $34y$ of the real test (section 4) and outputs at the three input locations ($25y$, $34y$ et $34x$). Predictions are made with 10 interface modes since it lead to the best results in frequency predictions.

The superposition in figure 8 of nominal, exact modified, and predicted modified FRFs show that the global effect of the modification is well represented. Only the error on the frequency of mode 4 leads to significant discrepancies.

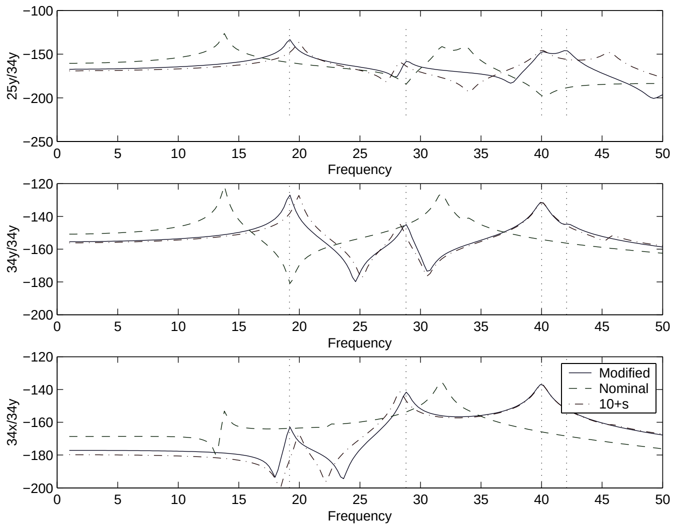

Figure 8: FRF for nominal, exact modified, modified with structural modification method.

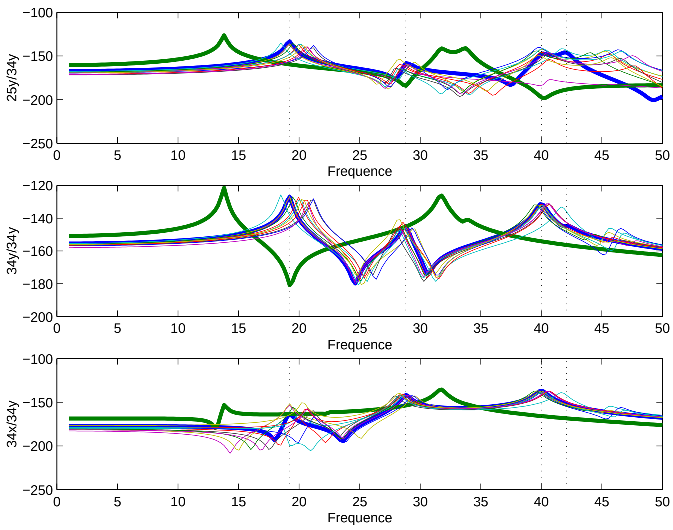

Figure 9: FRF for nominal (bold), exact modified (bold), and random realizations of approximate coupled prediction.

The next issue is the sensitivity of coupled predictions to errors in the base model. To evaluate this sensitivity, one considers an additive error of 10% on modeshapes ($c_T\tilde{\phi}_j = c_T\phi_j + \Delta(c_T\phi_j)$ with $|\Delta(c_T\phi_j)| = 0.1\,|c_T\phi_j|$) and an error of 1% on frequencies.

Even though the assumed errors are very high and not realistic, coupled predictions, shown in figure 9, match the global trends of the modified FRF very well. The method thus seems rather robust to errors in the base model.

A final issue is the ability to use partial tests where only a few modes are identified. For this purpose one only considers modes 3 and 4 for the base structure (31.3 et 33.7 Hz). The results shown in figure 10 are excellent in the neighborhood of the modes 3-4 for the modified structure. This of course is possible because the shape of this modes does not change very much with the modification. But again it is a reassuring check on the validity of the method.

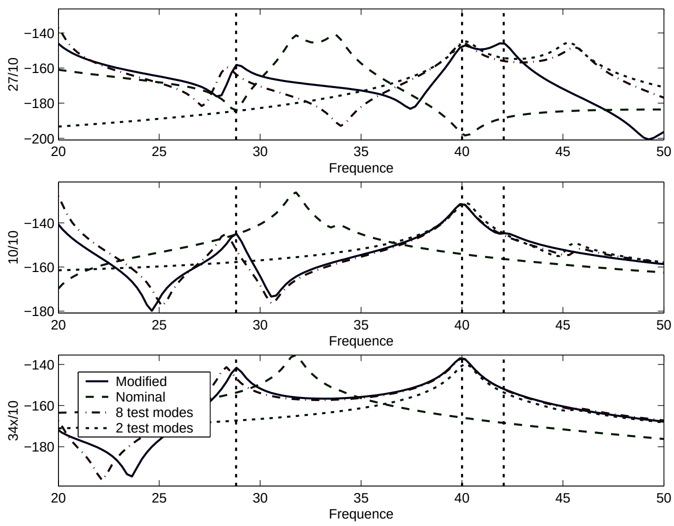

Figure 10: FRF for nominal, exact modified, and coupled prediction using 8 and 2 modes of the base structure.

4 APPLICATION TO A REAL TEST

4.1 Test data

The analytical example of section 3 was chosen to be somewhat similar to the true case shown in this section. In particular it used the same test geometry. The casing was tested by EDF/DER/AMV in 1998, the modification shown in figure 12 was proposed and built so that this gives a good practical example to test the proposed methodology.

The test were performed on site, with the casing coupled to the rotor, using hammer excitation. The test results are thus as good as can be expected in non laboratory conditions. The procedure uses reciprocity to build the modal model of the base.

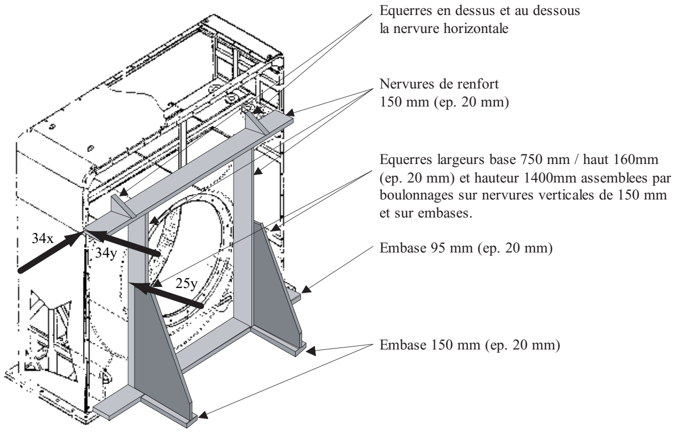

Figure 11: Final modification of the real casing.

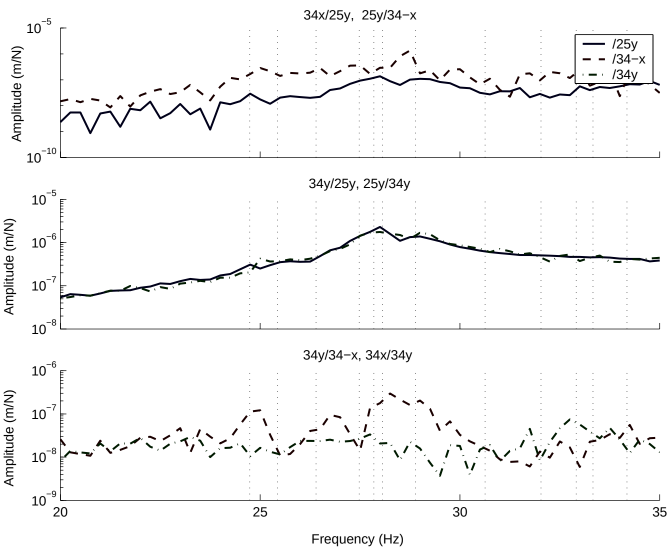

Figure 12: Reciprocity checks for nominal configuration.

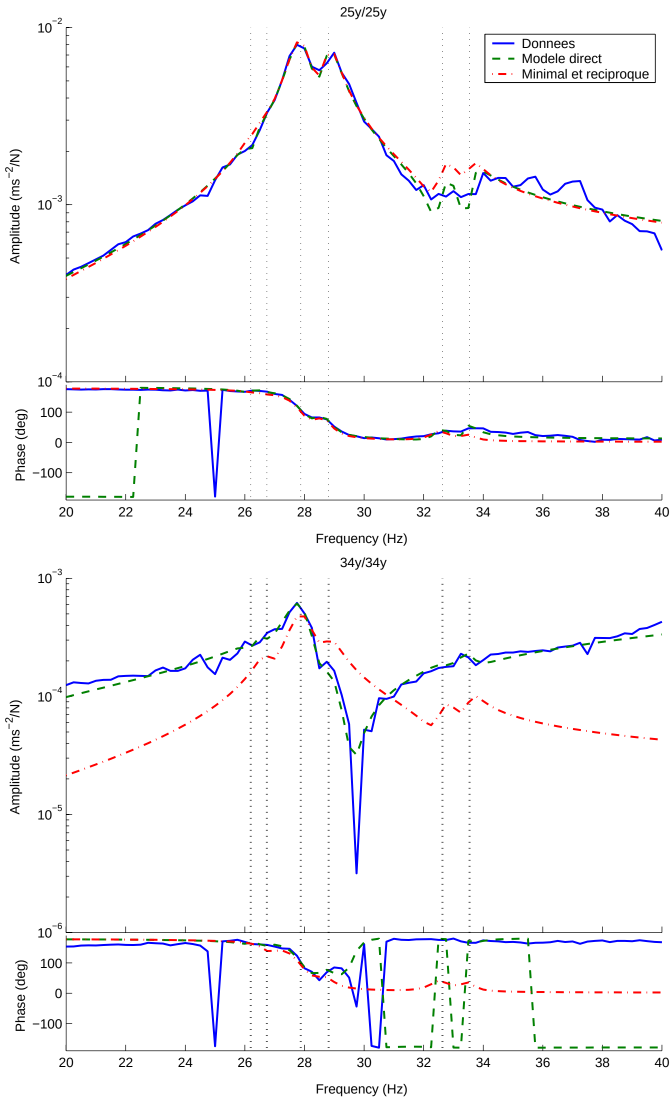

Figure 13: Test data, raw identification result and final base model with real modes verifying reciprocity.

The reciprocity check shown in figure 12 clearly indicates that the data does not verify the hypothesis very well particularly checks with 34x input/output.

In figure 13, on sees that the second order modal model of form (4) is quite good for the first driving point FRF (25y/25y) but significantly different for 34y/34y. Despite multiple trials, all attempts to build a reciprocal test model that would represent 34y/34y correctly failed. Since the raw identification result [14], which does not verify reciprocity but includes residuals, fits the data very well, this difficulty can be attributed to the high contributions of modes not taken into account in the modal model.

Despite this clear limitation of the modal model of the base structure a coupled prediction was made. The results show in figure 14 are very encouraging. The resonance occurs at 36 rather than 32 Hz, but the decrease in the 25 Hz region is well predicted for the 25y/34y FRF.

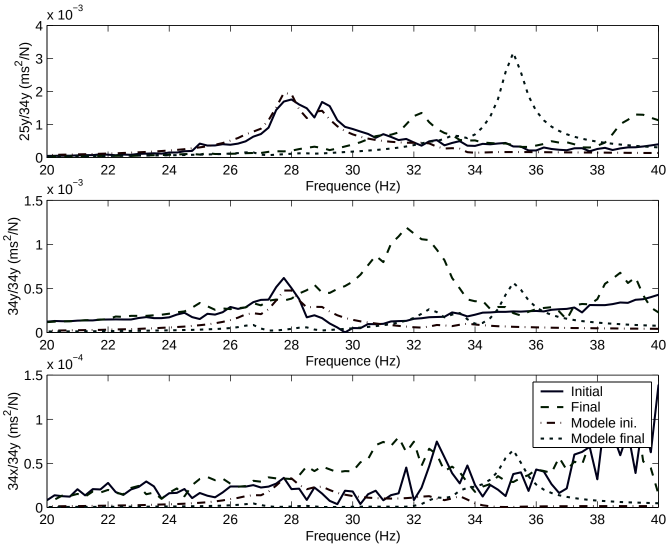

Figure 14: Initial and modified responses (test and analysis).

5 CONCLUSION

The main evolutions from standard structural modification methods are the use of a much simplified FEM model of the base structure to allow expansion and the use of generalized interface coordinates. Two tools have proven to be useful to obtain an *a priori* evaluation of the ability to obtain valid predictions.

- The computation of the first mode with sensors fixed which gives a good indication of the type of motion that cannot be represented with the chosen sensor configuration.
- The simultaneous use of more than one expansion method leading to various estimates of the interface motion which should be coherent for cases where good predictions can be obtained.

This first application obviously leaves much room for improvements of the procedure and, an even more crucial point, of the tools used to evaluate its validity on a given case without needing a detailed model of the structure under test. But results clearly demonstrate that the proposed methods have the potential to solve a much broader class of structural modification problems than previously thought possible.

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