A priori verification of local FE model based force identification.

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Abstract

The paper revisits force identification methods. Rather than using the classical inversion of the estimated transfers, one prefers to use a local model to generate an estimator of the modal states. The pseudo inverse in the force estimation process is then limited to the well-conditioned inversion of the dynamic stiffness in modal coordinates. The local model is also used to define physical loads rather than using the classical equivalent loads at sensors. The proposed strategy is illustrated on a numerical test case representing a pump motor. The objective of such simulations is not only to validate the methodology but also to show how the ability to identify forces should be validated *a priori*. Such validations can be performed for a given sensor configuration or used as the objective guiding a sensor placement algorithm that is presented.

1 Introduction

Experimental modal analysis techniques have become widely used for FE model updating, structural dynamic modification methods, stress analysis or structural monitoring. Normal modes, eigenfrequencies and damping can be estimated accurately. Knowledge of input forces is however necessary to obtain predictions of in-operation response. Many force estimation techniques have been developed in the time (see [1, 2] for example) and frequency domains (see [3, 4]). These techniques are based on in-operation measurements and a behaviour model of the structure under test.

Classical methods using generalized inverse techniques are presented in the first section. Estimation of dynamic loads acting on a structure is limited by the assumption that loads are applied at tests points. In many cases, where complex mechanisms are involved, this approximation is not fully relevant and would lead to erroneous predictions of in-operation behaviour, especially when a structural dynamic modification is to be tested. Some improvements based on the limitation of input forces locations have been developed to provided a better estimate [3]. Nevertheless, since generalized inverse methods deal with poorly conditioned problems, estimates are very sensitive to *a priori* chosen input positions. Moreover, forces are still assumed to act at sensor location.

When a tuned FE model is available, various techniques can be considered [5, 6] to achieve a good estimation of input forces. However, due to economic and technical constraint, building a tuned FE model of the tested structure is not always practical. To tackle these difficulties, section 3 motivates the need for a geometrical local FE model of the structure. This FE model is not tuned and only aims to create a strong kinematic link between *a priori* defined loaded interfaces and measurement points. This technique was successfully used in the structural dynamic modification field [7, 8] and is extended here to allow identification of distributed forces at non instrumented locations. The proposed improvements aim to take advantage of non-specific in-operation tests where actual load locations may not be instrumented. Model reduction and expansion techniques are used to overcome the difficulties linked to the limitations in sensor configuration and allow general definitions of loaded interfaces. The proposed methodology provides both a an accurate procedure to identify acting forces from given measurements, and practical tool to verify *a priori* sensor configurations and their ability to allow proper input force estimation.

Section 4 illustrates the complete methodology. It is applied on a numerical example selected to show the limitations of classical input force identification techniques. In-operation analyses are realistically simulated using no measurements at input force locations. Various test setups are introduced to discuss optimal sensor location and the accuracy of results is discussed.

2 Generalized inverse method

Generalized inverse methods are based on the initial determination by a modal test of a transfer function matrix $[H(\omega)]$ relating measurements $\{y_t(\omega)\}$ and inputs $\{u(\omega)\}$ by

$$\{y(\omega)\} = [H(\omega)]\{u(\omega)\} \tag{1}$$

In a second phase, the same structure is subjected to unknown loads and in operation measurements are made at the sensors. Assuming the linearity of the structure under test, the stationary of acting forces, and equivalent inputs to act at known input locations, one can estimate equivalent inputs $\{u_o(\omega)\}$ by solving

$$\{u_o(\omega)\} = \underset{\{u\}}{\text{ArgMin}}\left(|[H(\omega)]\{u\} - \{y_t(\omega)\}|^2\right) \tag{2}$$

In practice, however $\{y_t(\omega)\}$ is often not a statistically stable quantity. Thus unless all measurements are taken simultaneously, the critical information on the relative phase at different sensors is lost. Practical formulations of force identification, thus seek to estimate input auto- and cross-spectra based on output spectra. For given signals $\{a(\omega)\}$ and $\{b(\omega)\}$, the inter-spectra matrix $[G_{ab}]$ is defined by

$$[Gab(\omega)] \quad = \quad \{a(\omega)\}\{b(\omega)\}^H, \tag{3}$$

$$\tag{4}$$

where $[G_{ab}]$ is often obtained by averaging over a number of acquisition frames.

Classical relations between inputs and outputs of a linear filter $[H_t]$ give

$$[H_t][Guu_t(\omega)][H_t]^H = [Gyy_t(\omega)]. \tag{5}$$

Thus, assuming that the output spectra matrix $[G_{yy}]$ is estimated, the generalized inverse method based on equation (5) consists in finding $[Guu_o(\omega)]$ solution of

$$[Guu_o(\omega)] = \underset{[Guu]}{\text{ArgMin}}\left(\left|[H_t(\omega)][Guu][H_t(\omega)]^H - [Gyy(\omega)]\right|^2\right). \tag{6}$$

In the considered applications, the transfer functions are derived from test. In classical configurations with shaker or hammer inputs, only a few columns (or lines) of $[H]$ are known. Assuming reciprocity for the structure, along with normal modes and proportional damping allows to properly estimate the first normal modes $\{\phi_t\}$ and eigenvalues ω_t (see [9, 10]). An estimation of the full FRF matrix is then given by

$$[H(\omega)] \approx \sum_{k=1}^{N_{modes}} \frac{\{\phi_t\}_k \{\phi_t\}_k^T}{-\omega^2 + 2j\omega\omega_{tk} + \omega_{t_k}^2}. \tag{7}$$

The same formula can be used to estimate transfers for in operation modal tests and using the sensitivity method to scale mode shapes ([11]).

The spectral decomposition given in (7) shows that, at a given frequency, only few modes are involved in the dynamic response of the structure. Thus if $[H]$ is derived directly from test, its effective rank will be limited to that problems (2) and (6) are poorly conditioned (their solution is very sensitive to noisy measurements). If $[H]$ is given by (7) with identified modal properties, the problem becomes ill-posed if the number of potential inputs is higher than the number of modes (there exist various load configurations that have the same effect). J. O'Callahan and F. Piergentili ([3]) have thus demonstrated the high sensitivity of estimated forces to *a priori* chosen input location for a numerical test case with concentrated forces.

It is thus necessary to impose restrictions on potential inputs to obtain a well posed problem. Classically, force identification methods have assumed that possible loads were applied at sensors. P. Guillaume & al. in [4] have proposed to minimize the number locations with non zero inputs. While good results are shown, this does not account for the fact that forces acting on the structure are rarely point loads at sensors. There is thus no particular reason for *equivalent* forces at sensors to be localized on a few sensors.

The approach proposed in this paper is to use a coarse FE model to generate equivalent sensor or modal loads for physically defined loads (for example distributed loads near the bearings of a rotating machine or distributed pressure loads). This thus allows representing accurately physical loads even if they are localized at positions where no sensor could be placed. Based on this physical representation of loads, one can then expect to exploit force identification results directly.

3 Model based force estimation

To validate sensor placement *a priori*, account for distributed location or estimate input forces where no sensor is available, a FE model of the tested structure must be introduced. Effects of *a priori* defined inputs are computed all over the FE DOF and compared with in-operation data measured at sensors to best estimate loads. Similar principles were developed in [6] (boundary condition identification from modal analyses) and in [5] (identification of dynamic pressure in a cavity). One seeks to use similar ideas here with one less requirement : the FE model to be used should not need to be tuned. Extensive use of data expansion and model reduction techniques alleviates the need for mechanical consistency of the local model.

In the following on first introduces the local model of the structure, discusses its goals. Relevant reduction bases and force estimation are then discussed.

3.1 Local Model

Following the principles presented in [7], a coarse FE model of the structure, denoted *local model*, is introduced. The purposes of the *local model* are to ensure a mechanical relationship between the measurement points and considered inputs. A typical example of the use of a *local model* is presented in figure 1.

The structure is a motor pump group. In-operation vibrations mainly derive from internal out of balance forces. However, internal machinery is far too complex to be accurately modelled. Thus, the mesh of the local model is only based on external geometry, mechanical properties only ensures good regularity for computed displacement fields.

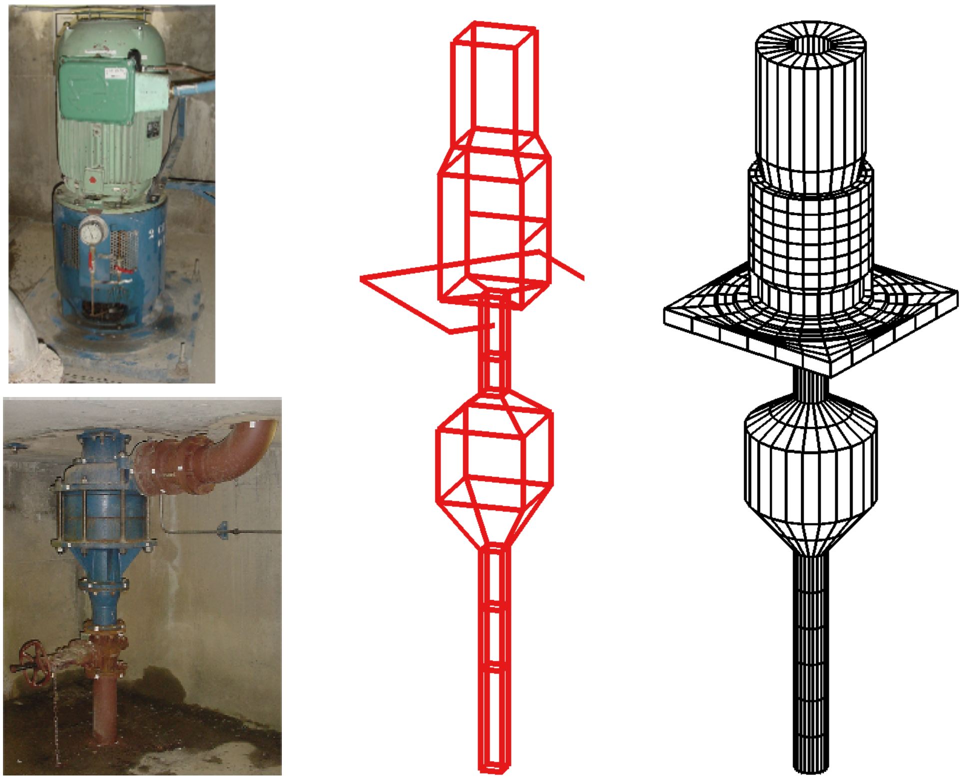


Figure 1: Sample local model : actual structure (left), test mesh (middle), and *local model* (right).

This coarse model is based on the overall geometry of the tested structure or sub domain, and includes some rough mechanical properties. The purpose is not to build a tuned FE model of the whole structure, but a geometrical model to interpolate measured displacements.

In this model, a clear distinction must be made between measurements ($\{u_t\}$ and $\{y_t\}$) and DOF ($\{q_M\}$) of the *local model*. Thus one has :

$$\left\{\begin{array}{rcl} [Z_M(\omega)]\{q_M(\omega)\} & = & [B_{Mt}]\{u_t(\omega)\}, \\ \{y_t(\omega)\} & = & [C_{tM}]\{q_M(\omega)\}. \end{array}\right. \tag{8}$$

Introduction of commandability matrix $[B_{Mt}]$ allows the definition of generalized forces, either distributed or concentrated. In this representation, the vector of inputs $\{u_t\}$ is *physical* (independent of the model) while $[B_{Mt}]$ describes the spatial effect of the *physical* inputs on the chosen model.

Combining equation (8) and relationships (3) leads to

$$[Gyy(\omega)] = [C_{tM}][Z_M(\omega)]^{-1}[B_{Mt}][Guu(\omega)][B_{Mt}]^T[Z_M(\omega)]^{-H}[C_{tM}]^T. \tag{9}$$

Since we are considering a coarse model, $[C_{tM}][Z_M(\omega)]^{-1}[B_{Mt}]$ is not an accurate representation of the actual transfer. The next step is thus to introduce test derived modal properties to replace the coarse model transfer while retaining the commandability matrices used to define physical loads.

3.2 Reduction basis

In-operation measurements mostly involve low frequency problems. Thus, the response is nearly contained in the subspace spanned by the modes of the considered frequency band. One can thus approximate the response of the whole structure using few of its modal coordinates $\{\eta_r\}$. Assuming modes of the whole structure defined on the local model given by $[\Phi_{Mr}]$, one thus has

$$\{q_M(\omega)\} = [\Phi_{Mr}]\{\eta_r(\omega)\}. \tag{10}$$

and the responses at sensors are given by

$$\{y_t(\omega)\} = [C_{tM}][\Phi_{Mr}]\{\eta_r(\omega)\}. \tag{11}$$

The *local model* is not a tuned FE model and measurements provide only a partial knowledge of ”true” mode shapes. It is thus proposed to use a subspace expansion method to estimate test mode shapes $[\phi_t]$ on the *local model* DOF. The subspace $[T_{Mr}]$ must ensure a proper representation of physical forces $\{u_t(\omega)\}$ and provide a good observation of test motion. Using the approach first introduced in [8], one considers the a first subspace generated by $\left[\hat{T}_{Mr}\right]$ verifying

$$[Z_M(\omega=0)]\left[\hat{T}_{Mr}\right] = \left[[B_{Mt}] \quad [C_{tM}]^T\right]. \tag{12}$$

The subspace defined by (12) provides more vectors than sensors, thus leading to ill conditioned expansion process. In order to select an appropriate basis, assuming that only low frequency problem are of interest, vectors of $\left[\hat{T}_{Mt}\right]$ are rearranged with respect to low strain energy. Hence, the expression of the reduction basis $[T_{Mt}]$ is

$$[T_{Mr}] = \left[\hat{T}_{Mr}\right][\phi_r], \tag{13}$$

where the vectors of $[\phi_r]$ are solution of the eigenvalue problem

$$\left(\left[\hat{T}_{Mr}\right]^T\left[\hat{K}_M\right]\left[\hat{T}_{Mr}\right] - \omega_r^2\left[\hat{T}_{Mr}\right]^T\left[\hat{M}_M\right]\left[\hat{T}_{Mr}\right]\right)\{\phi_r\} = \{0\}. \tag{14}$$

For regularisation issues, less vectors of $[\phi_r]$ than sensors are kept. Indeed, both experimental modal analysis and in-operation measurements results may be biased, due to noisy environment, small nonlinearities in structures, coupled phenomena, etc. Hence, smoothing is obtained by considering a much smaller subspace than the total amount of sensors.

Thus, let denote $\left[\hat{\Phi}_{Mr}\right]$ the expanded mode shapes. Vectors of $\left[\hat{\Phi}_{Mr}\right]$ are expected to verify

$$[\phi_t] \approx [C_{tM}]\left[\hat{\Phi}_{Mr}\right] = [C_{tM}][T_{Mr}][W_{rr}]. \tag{15}$$

Hence, columns of $[W_{rr}]$ are obtaind by solving the least square problems

$$\{W_{rr}\}_k = \underset{\{w\}_k}{\text{ArgMin}}\left(\left|[C_{tM}][T_{Mr}]\{w\}_k - \{\phi_t\}_k\right|^2\right) \quad \forall k \in [1 \,..\, N_{modes}], \tag{16}$$

3.3 Force identification using expanded shapes

Assuming that cross-spectra matrix $[G_{rr}(\omega)]$ for modal coordinates $\{\eta_r(\omega)\}$ is known, and considering equations (10) along with equation (5), one may write

$$[Z_{rr}(\omega)][G_{rr}(\omega)][Z_{rr}(\omega)]^H = \left(\left[\hat{\Phi}_{Mr}\right]^T[B_{Mt}]\right)[Guu]\left(\left[\hat{\Phi}_{Mr}\right]^T[B_{Mr}]\right)^H, \tag{17}$$

where

$$[Z_{rr}(\omega)] = -\omega^2\left[\backslash 1_\backslash\right] + 2j\omega\left[\backslash \omega_{t\backslash}\right]\left[\backslash \xi_{t\backslash}\right] + \left[\backslash \omega^2_{t\backslash}\right]. \tag{18}$$

Considering the definition of $\left[\hat{\Phi}_{Mr}\right]$, when more vectors of $[\phi_r]$ than expected entries in $\{u_t(\omega)\}$ are kept, $\left(\left[\hat{\Phi}_{Mr}\right]^T [B_{Mt}]\right)$ is full rank. Therefore, a direct estimation of $[Guu]$ is given by

$$[Guu] = \left(\left[\hat{\Phi}_{Mr}\right]^T [B_{Mt}]\right)^{\dagger} \left([Z_{rr}(\omega)] [Grr] [Z_{rr}(\omega)]^H\right) \left(\left(\left[\hat{\Phi}_{Mr}\right]^T [B_{Mt}]\right)^{\dagger}\right)^H . \tag{19}$$

This estimate of input spectra requires an estimate of the modal cross spectra $[G_{rr}]$. Looking at equation (11), generalized DOF $\{\eta_r\}$ can be defined as the solution of the least square problem

$$\{\eta_r\} = \underset{\{\eta\}}{\text{ArgMin}} \left(\left|[C_{tM}] \left[\hat{\Phi}_{Mr}\right] \{\eta\} - \{y_t\}\right|^2\right). \tag{20}$$

Since the projection of the reduction basis $\left[\hat{\Phi}_{Mr}\right]$ on the sensors corresponds to identified modes, $[C_{tM}] \left[\hat{\Phi}_{Mr}\right]$ is full rank. Thus, one has

$$\{\eta_r\} = \left([C_{tM}] \left[\hat{\Phi}_{Mr}\right]\right)^{\dagger} \{y_t\}. \tag{21}$$

Hence, cross sprectra associated to generalized DOF $\{\eta_r\}$ are given by

$$[G_{rr}(\omega)] = \left([C_{tM}] \left[\hat{\Phi}_{Mr}\right]\right)^{\dagger} [Gyy] \left(\left([C_{tM}] \left[\hat{\Phi}_{Mr}\right]\right)^{\dagger}\right)^H . \tag{22}$$

4 Numerical test case

A realistic test case has been set up to illustrate the proposed methodology. The first section introduces the structure. It represents an industrial case of typical application. Emphasis is put on numerical simulation, in order to model true testing conditions. The second section introduces a practical sensor placement tool to improve a given configuration, and presents the results for various test set-ups.

4.1 Numerical structure & simulation

The test case aims to be representative of a machine where In-operation vibrations are mainly due to internal excitation, such as rotating machines or piping. Moreover, internal excitations can be distributed (pressure, turbulent flow). As a consequence, no sensor can be placed where forces act, but *a priori* assumptions can be defined, such as preferred locations (i.e. bearings in rotating machines, faucets in piping, etc.) and *a priori* spatial distribution.

The FE model of test structure used for the simulation is presented on the figure 2. The *local model* is also presented. It is based on the same geometry, except it has been simplified, and exhibits different boundary conditions.

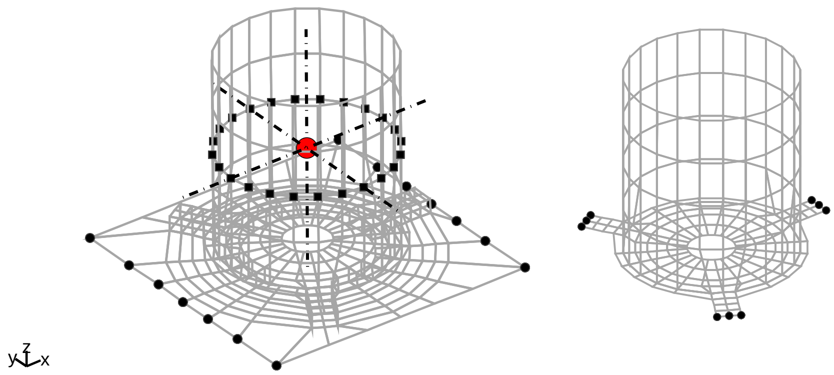


Figure 2: Numerical structures - Complete FE model showing input forces locations (left) and *local model* (right)

Internal excitation is assumed to be applied at the centre of the cylinder, depicted with a red dot. Forces are transmitted to the structure by the mean of linear constraints involving FE DOF, depicted with black squares. It is assumed that displacements of input force locations are weighted sums of transmitting FE DOF displacements. Boundary conditions (all clamped) are depicted using circular markers.

Input forces contain both harmonic and large bandwidth excitation, and are defined by

$$\left\{\vec{f}\left(t\right)\right\} = \left\{ \begin{array}{c} f_x\left(t\right) = \left(\sum_{i=1}^{N_{harm}} sin\left(\omega_i t + \varphi_i\right)\right) + \hat{x}\left(t\right) \\ \\ f_y\left(t\right) = \left(\sum_{i=1}^{N_{harm}} cos\left(\omega_i t + \varphi_i\right)\right) + \hat{y}\left(t\right) \\ \\ f_z\left(t\right) = 0 \end{array} \right\}, \tag{23}$$

where $\hat{x}\left(t\right)$ and $\hat{y}\left(t\right)$ are noises of given power spectral density.

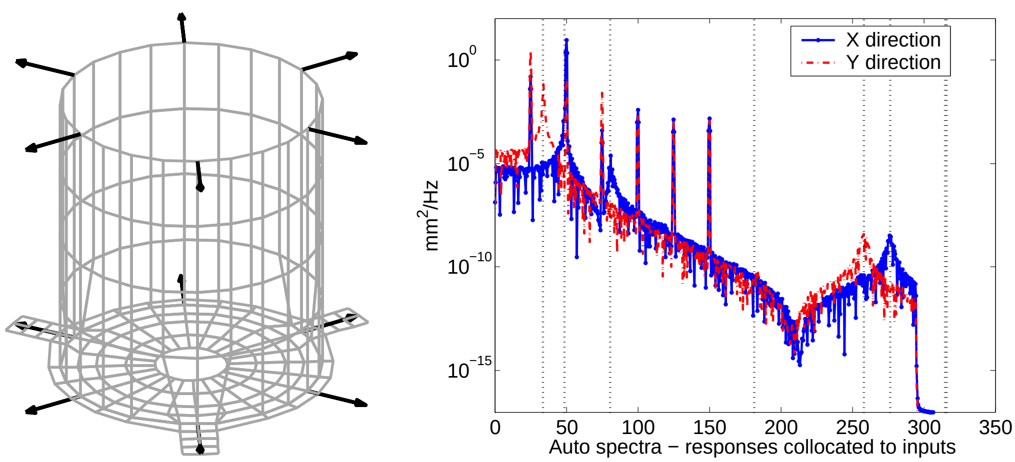


Figure 3: Test configuration - *Local model* with fine manual sensor configuration (left) and displacement power spectra collocated to inputs (right).

A realistic simulation is first performed, to validate the complete methodology. A 50 second time response of the structure to inputs is computed. Signal processing is carried out fully, so that representative averaged cross spectra matrices can be used to estimate inputs forces cross spectra. Sensor configuration and response spectra are presented figure 3. Eigen frequencies of the FE model are shown using vertical dotted lines. Sensor configuration is quite fine. This first case aims to demonstrate the feasibility of force identification in real test conditions.

A modal model is built using all modes involved in the response, that is to say the first six modes. To perform a good reconstruction of overall displacement, the first ten modes of $[T_{Mr}]$ are used. Input estimation results are presented on the figure 4.

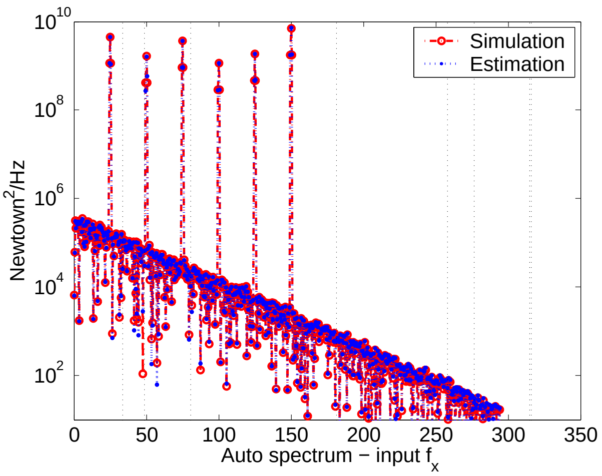


Figure 4: Auto spectrum results - $f_x(\omega)$ (left) and $f_y(\omega)$ (right).

Despite the very noisy data, results are very accurate. Both harmonic peaks and large bandwidth noise are well estimated. *Local model* used is not an accurate model of base structure, since it uses different properties and boundary conditions, but it provides a pertinent basis for expansion process. This first case thus demonstrates the consistency of previously described concepts.

4.2 Sensor placement

In force identification problem, sensor configuration is of critical importance to perform a proper estimation. An objective here is to propose a simple practical tool to improve sensor placement. This tool allows the optimal placement of additional sensors, starting from a given configuration, or from scratch. The underlying principle is to place the new sensor at the location of the maximum response to the considered inputs with the already placed sensors fixed. One thus solves

The response $[q_M]$ of the whole structure, under these assumptions, is given by

$$\left\{\begin{array}{l}[Z_M]\,[q_M]=[B_{Mt}]\\ [C_{tM}]\,[q_M]=\{0\}\end{array}\right. \tag{24}$$

When multiple entries are defined in $[B_{Mt}]$, a maximum response is given through the singular value decomposition. Thus, one has

$$[q_M]=[U]\,[S]\,[V]^H\,. \tag{25}$$

The observation matrix with new sensor (the procedure is restarted for every new sensor) is given by

$$[C_{tM_{up}}]^T = \left[[C_{tM}]^T \quad \{C_{add}\}^T \right], \tag{26}$$

where $\{C_{add}\}$ corresponds to the observation of DOF with the maximum response in the left eigenvector associated to the smallest singular value,

$$\{C_{add}\}_i = 0 \quad \forall i \neq j,\ j \parallel U_{j,1} = \operatorname*{Max}_k \left(U_{k,1} \right). \tag{27}$$

To check force estimation procedure and sensor placement tool, three configurations, presented figure 5, have been defined. The first one (config. A) is to be compared with the configuration defined in the section 4.1. Twelve sensors have been placed on the structure from scratch. Results are presented figure 6. Expansion basis is still built using the first ten vectors of $[T_{Mr}]$. Like in the case presented figure 4, results are very accurate. In both cases, very good results are obtained, so that no performance degradation due to sensor placement tool is shown.

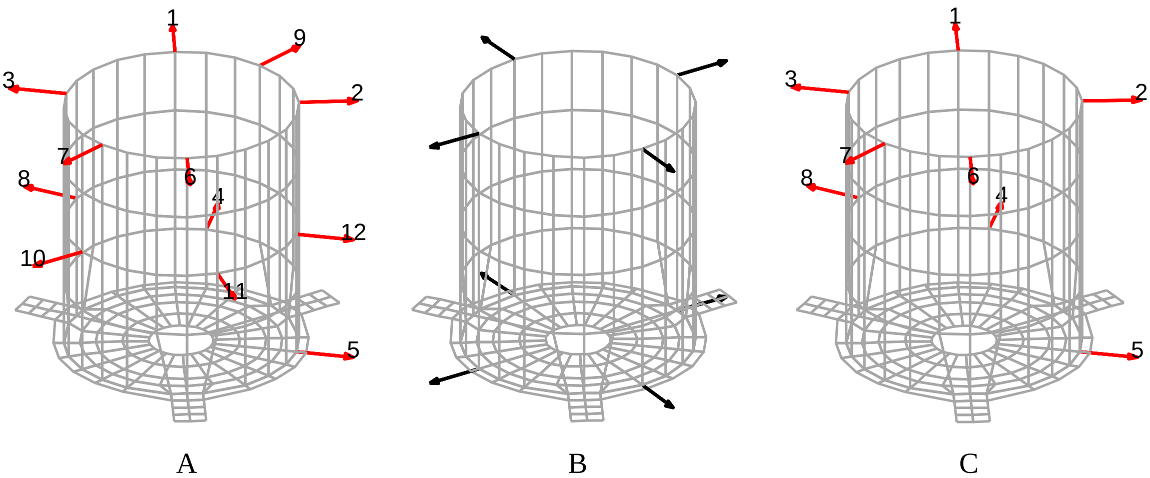


Figure 5: Test configurations : A - Automated placement - fine configuration
B - Manual placement - basic configuration
C - Automated placement - basic configuration

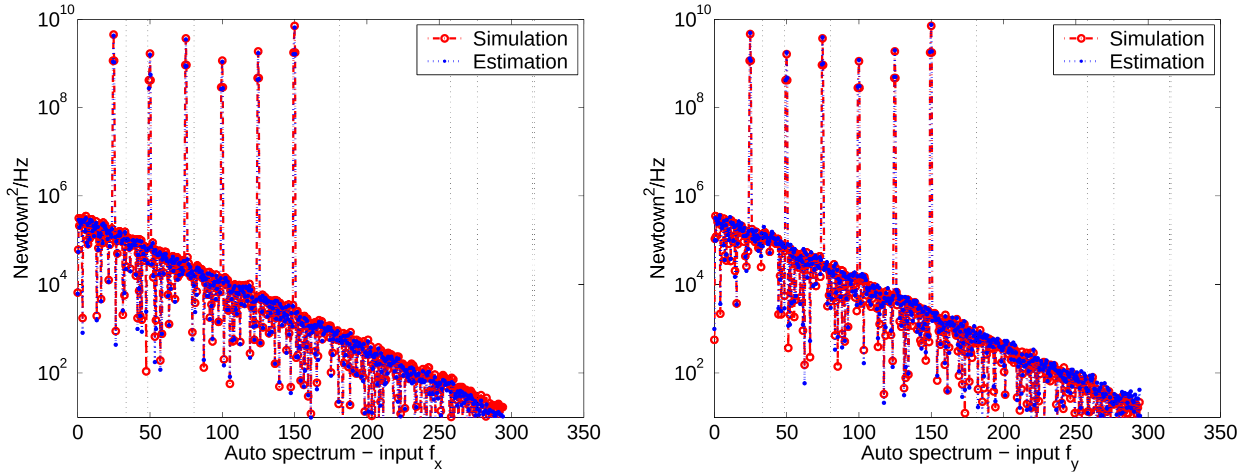
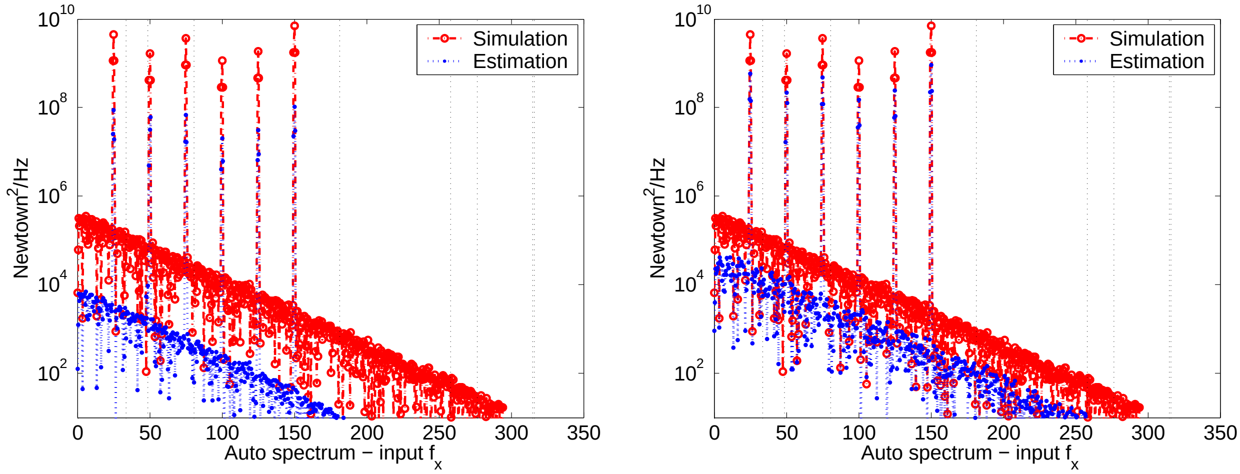


Figure 6: Auto spectrum results for configuration A - $f_x(\omega)$ (left) and $f_y(\omega)$ (right).

Sensor configuration provided using the proposed tool is quite pertinent and looks fine. Eigenmodes of the structure mostly involve swinging of the main cylinder. Automatically placed sensors are able to observe these deformations, and looks like a configuration that may have been defined manually. Placement order is noted on the figure 5.

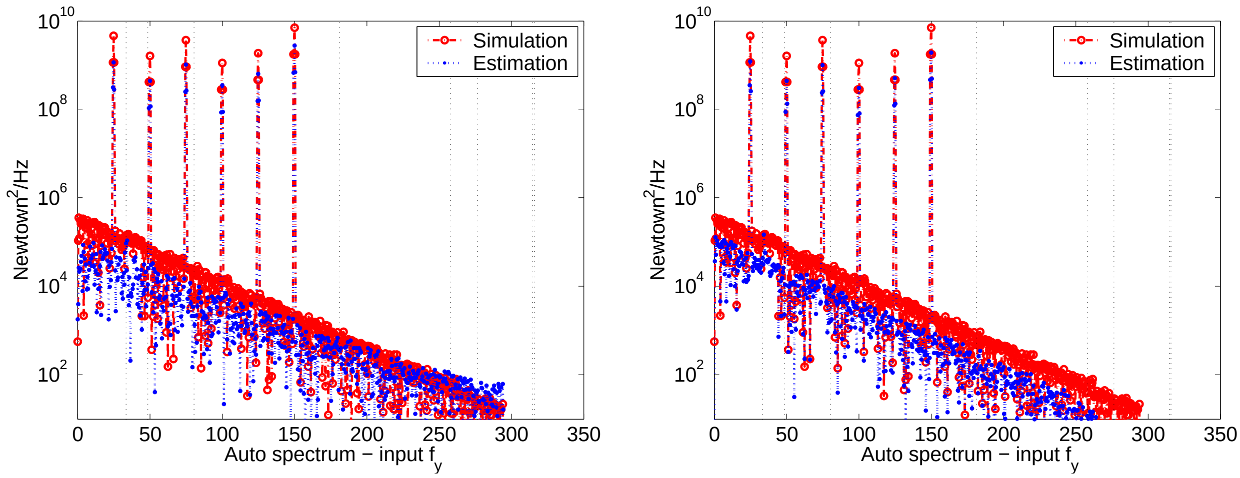
Since good force estimates are obtained, one seeks to analyse the effect of limiting the number of sensors to 8 while observing the first six modes of the structure. Configuration B corresponds to a basic manual placement, configuration C is an automated placement from scratch. Figures 7 and 8 show the results for estimated auto power spectra of $f_x(\omega)$ and $f_y(\omega)$ for both configuration configurations.



Configuration B

Configuration C

Figure 7: $f_x(\omega)$ - auto spectrum results.



Configuration B Configuration C

Figure 8: $f_y\left(\omega\right)$ - auto spectrum results.

Both configuration are rough, and do not allow a really accurate estimation of input forces. For configuration B, estimated inputs are five times lower than true values in the best case (f_y), and near a hundred times for f_x. Configuration C provides more homogeneous results. For both f_x and f_y, estimations are near ten times lower than effective inputs. But the balance between f_x and $_y$ is kept, hence facilitating the mechanical analysis of behaviour, allowing to identify force sources. The automated placement thus gives much better result than the manual one.

Another important result of the proposed methodology is the ability to use different sensor configurations for modal analysis and in-operation tests. Indeed, purposes of modal analysis and in-operation measurements may be quite different, thus leading to largely different placements to get accurate results. In classical methods, one was limited to the use of the same set of sensors. Here, data expansion and model reduction techniques can overcome the difficulty.

5 Conclusion

A original approach allowing both to estimate input forces and validate sensor placement for in-operation test is presented. A coarse FE model, called *local model*, is used to make hypotheses on input forces location, so that accurate sensor placement can be defined for optimal observability and reconstruction of input force spectra.

This *local model*, based on the geometry of the measurement sub domain, allows the non-coincidence of measurement points and input force locations. Thus both distributed forces and concentrated inputs can be identified. A smoothing expansion basis based on the eigenmodes of a reduced model derived from the *local model* is computed. Tests data are interpolated and the information concerning input forces is estimated. Based on the *local model*, a practical tool providing optimal sensor addition to a given configuration is also introduced.

A realistic numerical example, based on an industrial case study, is presented to illustrate some advantages of the proposed approach. A load, corresponding to out of balance forces (including harmonics peaks) added to a large bandwidth noise is taken into account. The ability of this method to reconstruct input forces spectra using few hypotheses is shown and the good performance of the proposed sensor placement algorithm is illustrated for various test set-ups.

The results shown are quite encouraging. The number of input forces, the definition of their *a priori* location, the properties and geometry of the local model are a few of the parameters whose influence needs to be evaluated in more detail. Experimental validations of the proposed methodology are also being considered.

Acknowledgements

The photographs on figure 1 were given by courtesy of Electricité de France, EDF R&D, CLAMART, FRANCE

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