MODEL REDUCTION FOR SYSTEMS WITH FREQUENCY DEPENDENT DAMPING PROPERTIES.

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ABSTRACT

Isolation devices made of polymers, tubes filled with propergol, distributed vibration damping treatments and a number of other structures have vibration characteristics that have a significant dependence on the properties of viscoelastic materials. It has been shown that such materials can be represented over broad frequency ranges by frequency dependent complex moduli. The use of such constitutive laws is theoretically simple but is not directly compatible with traditional spectral decomposition methods (modal projection methods) based on frequency independent matrices. Thus, for large models (with a few thousand DOFs) and/or large frequency bands (with many frequency points), the cost of an assembly and direct solution at each frequency point often makes the approach impractical.

The paper analyses the validity of reduced models obtained by projection on bases of real valued Ritz vectors. Issues addressed in particular are the importance of real poles, the ability to build dynamically equivalent models with frequency independent matrices expressed in principal coordinates, and the impossibility construct a model with frequency independent local properties that would have the same behavior. Examples treated are a truss with local isolators and a panel with a constrained viscoelastic layer.

1. INTRODUCTION

Proper modeling of the damped behavior is important in many applications. Polymer supports for isolation of turbine blades, engines or instruments [1], sandwich metal/viscoelastic/metal panels to limit acoustic emissions [2], rockets filled with propergol [3] are just a few examples.

Linear viscoelastic materials are characterized by complex, frequency dependent, constitutive laws giving a linear relation between stresses and strains [4-6]. For non linear systems (many dissipation mechanisms, friction, plasticity, shocks, etc. are non linear), the harmonic responses at a given amplitude give information similar to that of transfer functions. It is thus usual, and quite efficient, to build a viscoelastic representation equivalent to a non linear response by assuming that a measured or predicted harmonic response corresponds to the transfer function of an equivalent viscoelastic structure [7].

Since the stress/strain relationship remains linear, viscoelastic computations only differ from elastic ones by the need to support complex valued matrices. In many practical cases however, the dependence on frequency of the constitutive law is fairly complex so that the viscoelastic model must be assembled and solved for the response at each considered frequency. This can become excessively expensive even for models with only a few thousand degrees of freedom [13]. Even when viscoelastic predictions are affordable, there does not usually exist a simple way to construct an equivalent time domain representation of the model [3], so that transient analysis is not possible.

Basic viscoelastic constitutive laws, dynamic stiffness and flexibility computations, and truncation of partial fraction descriptions of the response are discussed in section 2.

Section 3 then shows how methods considered for the reduction of undamped models lead to reduced damped models with a non linear dependence on frequency. Given a non linear reduced model, one then analyses different methods of the construction of dynamically equivalent second order models with frequency independent matrices. It is finally shown that such models are only meaningful in quasi-principal coordinates so that for many cases the determination of local damping properties from a global test is not a well posed problem.

2. BASIC PROPERTIES OF VISCOELASTIC MODELS

2.1. Constitutive laws and structural models

For materials, the most common rheological models for linear viscoelastic constants are shown in figure 1. Partial lists of other usual models can be found in Ref. [4-6].

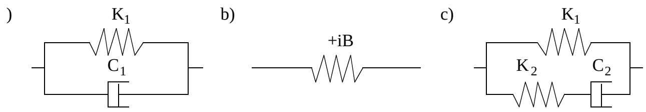

Fig. 1: Rheological models of damped solids: a) viscous damping, b) complex stiffness (structural damping), c) standard viscoelastic solid (3 parameter form).

Figure 2 shows the dependence on frequency of figure 1 models. Numerical values used here are $K_1 = 10K_2 = 1.5e5, C_1 = C_2 = 30$, $B=3000$. The real part corresponds to an elastic stiffness (stress amplitude to strain amplitude ratio) and the tangent of the phase to the loss factor (energy dissipated per cycle of harmonic excitation [7]). The viscous model is dominated by stiffness at low frequencies (no dissipation) and by damping at high frequencies (dissipation tends to infinity). The complex stiffness model gives a dissipation that is independent of frequency, which is a good approximation for many materials or links (welds, rivets, bolts). For many polymers however, the dependence of the dynamic stiffness on frequency is significant and only more complex models such as that of the standard viscoelastic solid give an appropriate representation. Such constitutive laws are generally determined through careful testing of material samples.

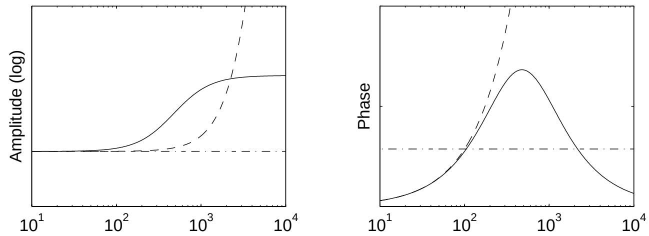

Fig. 2: Dynamic stiffness associated to standard rheological models of figure 1. (---) viscous, (·-·) structural and (—) viscoelastic damping.

Given a constitutive law for all the materials used, the finite element method allows the construction of models of the general second order form

$$\left[Ms^2+Cs+K+iB\right]\{q\}=[b]\{u\} \\ \{y\}=[c]\{q\} \tag{1}$$

where the dynamic stiffness is composed of a real part linked to mass and stiffness contributions (Ms^2+K) and an imaginary part combining the different dissipation mechanisms ($Cs+iB$). Depending on the constitutive laws, the so-called stiffness K, viscous damping C and structural damping B matrices are constant or frequency dependent.

In particular a model containing a standard viscoelastic solid will take the form

$$\left[Ms^2+K+K_v\frac{(1+\beta s)}{(1+\alpha s)}\right]\{q\}=[b]\{u\} \\ \{y\}=[c]\{q\} \tag{2}$$

which is really a third order model but can also be written in a second order form provided that additional degrees of freedom are used [8].

2.2. Frequency response functions

Linear dynamic systems are only characterized by their transfer functions, which for most applications in structural dynamics, characterize the relation between applied forces u and resulting displacements y. For models of the form (1) transfer functions are thus given by

$$\{y\}=[c]\left[Ms^2+Cs+K+iB\right]^{-1}[b]\{u\}=\left[\alpha_{bc}(s)\right]\{u\} \tag{3}$$

where it clearly appears that frequency dependent B and C matrices can have the same effect since only the sum $Cs+iB$ appears in the model.

Even for constant matrices, the distinction between different damping models can be difficult. Let us consider the example of a spring mass system whose spring would follow the rheological models of figure 1. For the parametric values $K_1=10K_2=1.5e5$, $C_1=C_2=30$, $B=3000$ and $M=15$, the resonance frequency is near 100 where the three models have the same loss factor (the dynamic stiffnesses have the same phase as shown in figure 2). Figure 3 clearly shows that the amplitude and phase responses of the three models are almost identical. The viscoelastic model shows a slight frequency shift near the phase transition which is visible as an offset in the Nyquist plot (figure 3b).

For this simple case, all three models thus result in almost perfectly equivalent dynamic flexibilities. This equivalence does not imply that the dynamic stiffnesses are equal, in particular the imaginary parts differ significantly as shown in figure 3d.

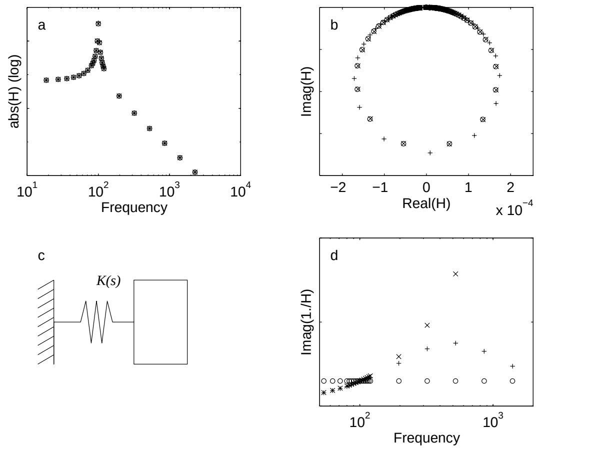

Fig. 3: **ab)** dynamic flexibility of a spring mass system **c)** with the spring following the dynamic stiffness of models shown in figures 1-2. **d)** dynamic stiffness in the 100-1000 rd/s range. (x) viscous (o) structural (+) viscoelastic damping.

2.3. Complex modes and partial fraction descriptions

The response of a linear system is usually associated to a partial fraction description, also called pole/residue parametrization,

$$H(s)=\sum_{j=1}^{\infty}\left(\frac{\{c\psi_j\}\{\psi_j^T b\}}{s-\lambda_j}\right) \tag{4}$$

as a sum of independent responses of complexes modes characterized by the modal input shape vectors $\{\psi_j^T b\}$, modal output shape vectors $\{c\psi_j\}$ and poles λ_j. Complex modes are non trivial solutions of the homogeneous response so that for a second order model of the form (1) they verify

$$\left[K(\lambda_j)\right]\{\psi_j\}=\{0\} \text{ with } [K(s)]=\left[Ms^2+Cs+K+iB\right] \tag{5}$$

For a real system one must have $H(\bar{s})=\overline{H(s)}$ from which it follows that real valued poles should be associated to real valued complex modes and complex valued poles should come in complex conjugate pairs with complex conjugate modes

$$\alpha(s)=\sum_{j=1}^{NC}\left(\frac{\{c\psi_j\}\{\psi_j^T b\}}{s-\lambda_j}+\frac{\overline{\{c\psi_j\}\{\psi_j^T b\}}}{s-\overline{\lambda_j}}\right)+\sum_{j=1}^{NR}\left(\frac{\{c\psi_{j\,\mathrm{Re}}\}\{\psi_{j\,\mathrm{Re}}^T b\}}{s-\lambda_{j\,\mathrm{Re}}}\right) \tag{6}$$

Except for wave models which are only applicable to very specific structures, the models considered are always finite. The response is therefore only modeled for a given frequency range. For a modeled range (gray area in figure 4a), the total response can be decomposed as the sum of modal and residual contributions

$$\left[\alpha_{\text{Total}}(s)\right] = \left[\alpha_{\text{Modal}}(s)\right] + \left[\alpha_{\text{Residual}}(s)\right] \tag{7}$$

In usual applications, the modal contributions correspond to real poles or pairs of complex conjugate complex poles whose frequencies are within the modeled bandwidth. In many cases, the model also contains other "correction modes" which give an approximation of residual contributions (figure 4b). In experiments, a frequency independent residual flexibility is often considered which corresponds the use of a correction mode with a pole at infinity.

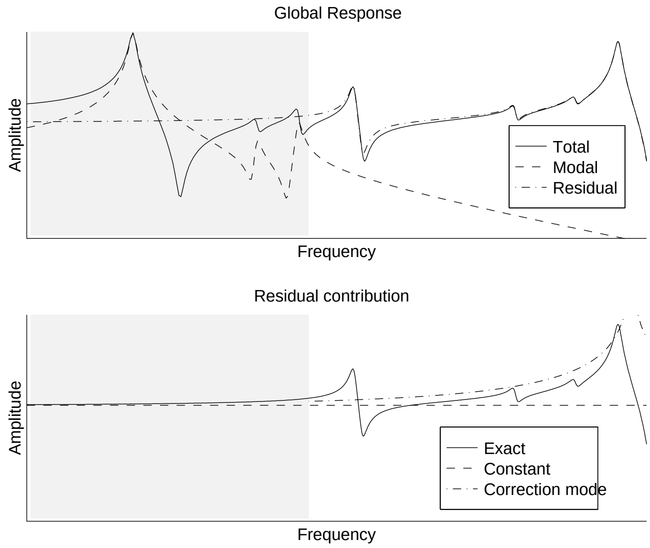

Fig. 4: a) decomposition of response in modal and residual contributions. b) approximation of residual response by a constant or an out of band correction mode.

A model of the form (1) usually contains both modal contributions and approximations of residual terms. If it accurately represents the true system response, it will usually be called complete. It is useful to note that modes of complete second order models verify a properness condition [9]

$$\sum_{j=1}^{NC} 2\,\text{Re}\left(\left\{\psi_j\right\}\left\{\psi_j^T\right\}\right) + \sum_{j=1}^{NR} \left\{\psi_{j\,\text{Re}}\right\}\left\{\psi_{j\,\text{Re}}^T\right\} = 0 \tag{8}$$

which is easily derived from the fact that the mass contribution dominates the dynamic stiffness at high frequencies so that, for arbitrary input b and output c shape matrices, the mobility tends to zero

$$\lim_{s\to\infty}\left(s\{y\}\right) = \lim_{s\to\infty} s[c]\left[Ms^2 + Cs + K + iB\right]^{-1}[b]\{u\} = 0 \tag{9}$$

It is important to note that the constraints introduced by the use of a complete model can be significant for systems with highly non proportional damping.

3. REDUCTION OF DAMPED FINITE ELEMENT MODELS

3.1. Reduction on bases of real valued vectors

Ritz approximations of a second order model of the form (1) assume that the response lies within a subspace of basis T ($\{q\} = [T]\{q_R\}$) and that the projection of the error is orthogonal to the dual subspace T^T. For a chosen basis T, the approximation of the model (1), called reduced model, is thus given by

$$\begin{gathered}\left[T^T M T s^2 + T^T C T s + T^T K T + i T^T B T\right]\left\{q_R\right\} = \left[T^T b\right]\{u\} \\ \{y\} = [cT]\,\{q_R\}\end{gathered} \tag{10}$$

Many methods have been devised for the construction of real projection bases. One will refer in particular to the literature on condensation [10] and Component Mode Synthesis [11]. Reduction bases usually considered combine normal modes and static corrections defined below.

For an analytical model with a constant stiffness matrix, normal modes are defined by the conservative eigenvalue problem

$$-[M]\left\{\phi_j\right\}\omega_j^2 + [K]_{N\times N}\left\{\phi_j\right\}_{N\times 1} = \{0\}_{N\times 1}, \tag{11}$$

where the mass is symmetric positive definite, the stiffness symmetric positive semi-definite and there are N independent eigenvectors ϕ_j (forming a basis noted ϕ) and eigenvalues ω_j^2 (forming a diagonal matrix noted Ω^2). As solutions of the eigenvalue problem (11), the N eigenvectors verify to necessary and sufficient orthogonality conditions with respect to mass and stiffness

$$[\phi]^T[M][\phi]_{N\times N} = \left[\backslash \mu_{\backslash}\right]_{N\times N} \text{ and } [\phi]^T[M][\phi] = \left[\backslash \mu_{\backslash} \backslash \Omega^2_{\backslash}\right] \tag{12}$$

where the modal masses μ are arbitrary and set to unity ($\mu = I$) in the rest of this paper.

For a finite element model, one only computes the low frequency normal modes but for all applications one also restricts the number of applied forces to be considered. For a set of forces described by the frequency independent matrix b, the static response is given by

$$T_A = [K]^{-1}_{N\times N}[b]_{N\times NB} = \sum_{j=1}^{N} \frac{\phi_j \phi_j^T b}{\omega_j^2} \tag{13}$$

where the vectors T_A are generally called *attachment modes* [11] and alternate definitions are given for structures with rigid body modes. Applied loads can also be defined in terms of imposed displacements at certain DOFs which naturally lead to *constraint modes* which form in many cases a basis equivalent to that of the attachment modes [11].

As mentioned in section 2 frequency dependent constitutive laws lead to a frequency dependent complex stiffness K. Following the multimodel reduction approach proposed in Ref. [12-13], one can consider the bases resulting from the reduction linked to different stiffness values (model characterized by M and real part of $K(i\omega_a)$ at different values of ω_a). The different bases thus obtained can then be combined to obtain the final projection basis ($T = \begin{bmatrix} T(\omega_a) & T(\omega_b) & \cdots \end{bmatrix}$) with the possibility of selecting principal directions in this basis (as discussed in Ref. [14]).

Let us consider the truss structure shown in figure 5a where the upper beam is supposed to be connected to the lower support truss by 4 low stiffness isolators following a 3 parameter viscoelastic damping law (see more details on this example in [15]). In figure 5b, the model clearly contains lightly damped poles which correspond to the usual structural modes and real poles linked to the presence of the viscoelastic dampers. The real poles are all very close to 1000 rd/s which is the frequency of the real pole of the 3 parameter modulus model.

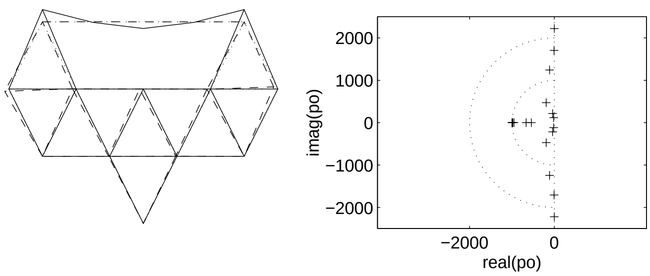

Fig. 5: **a)** first payload bending and first support bending modes (details of the example in [15]). **b)** position of poles in the complex plane with equal frequency circles.

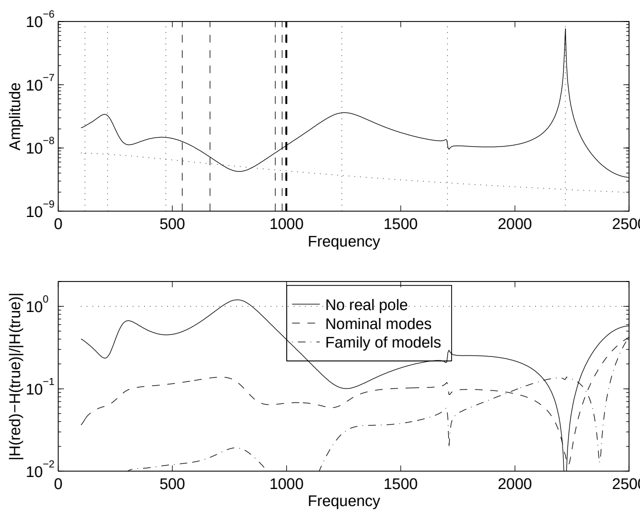

Fig. 6: **a)** frequency response with (····) contribution of real poles. **b)** quality of FRF predictions indicated by $\left|H_{Model} - H_{True}\right| / \left|H_{True}\right|$ which should be much smaller than (····) 1.

In the frequency response function shown in the figure 6a, the frequencies of the real poles linked to the viscous dampers are within the model bandwidth (vertical dashed lines). The contributions of the real poles are significant (dotted line) so that in figure 6b the model with the real pole contributions truncated is really poor ($\left|H_{Model} - H_{True}\right| / \left|H_{True}\right|$ higher than 1 at low frequencies). Many systems have much lower contributions of real poles which allows the approximations by frequency independent matrices discussed in section 3.2.

The other two models shown in figure 6b correspond to

- a reduction on the basis of the normal modes of the nominal structure (low frequency modulus) corrected with the attachment mode associated to the considered input and
- a reduction using the two sets of normal modes based on two values of the modulus (with selection of principal directions to compare two reduced models of the same size).

The second approach only improves the model quality at in the lower part of the frequency range. This illustrates the fact that reductions based on families of models rarely give less accurate results than nominal reductions but may occasionally not improve the prediction very much.

3.2. Equivalent frequency independent reduced models

In the reduction process, the frequency dependence of the viscous damping and complex stiffness matrices is retained, thus leading to an input/output model of the form

$$\{y\} = [cT]\left[K_R(s)\right]^{-1}\left[T^T b\right]\{u\} = \left[\alpha(s)\right] \{u\} \qquad (14)$$

where the dynamic stiffness $K_R(s)$ is typically a fairly complex function of frequency.

For predictions, one is interested in determining a simpler model with frequency independent mass, stiffness and viscous or structural damping matrices. The simple spring mass example of section 2.2 indicated that this might be possible, extensions to more complex problems are considered here.

The initial idea would be to seek an approximation of the dynamic stiffness $K_R(s)$

$$\left[M_E, C_E, K_E\right] = \arg\min\left\| M_E s^2 + C_E s + K_E - K_R(s)\right\| \qquad (15)$$

In such approximations, the choice of the matrix norm and of retained frequencies is however far from obvious. The simplest approach is to retain a single dominant frequency and approximate the matrix term by term. This is for example used in NASTRAN [16] to build equivalent viscous damping models form initial models with structural damping

$$C_E = \frac{1}{\omega_D} B \qquad (16)$$

This approach is not however directly applicable to cases where the properties are frequency dependent. Two approaches were considered here. The first uses evenly spaced frequencies in the considered frequency band and seeks, term by term, a least square approximation of the dynamic stiffness

$$\begin{aligned} \left[(M_E)_{ij}, (K_E)_{ij}\right] &= \arg\min\left\| \left(M_E s^2 + K_E - \mathrm{Re}(K_R(s))\right)_{ij}\right\|_2^2 \\ \left[(C_E)_{ij}\right] &= \arg\min\left\| \left(C_E s - \mathrm{Im}(K_R(s))\right)_{ij}\right\|_2^2 \end{aligned} \qquad (17)$$

The second approach tries to define a frequency of interest for each term of the dynamic stiffness. To do so, it starts by projecting the dynamic stiffness on a nominally orthonormal basis (i.e. $T^T M T = I_{NR}$ and $T^T K_0 T = \left[\backslash \Omega_{R\backslash}^2 \right]$ where the Ω_R are the normal mode frequencies of the reduced model which may differ from those of the full order model).

Assuming that the modes do not change when the non nominal viscoelastic modulus is used, the passage at zero of the diagonal terms of $K_R(s)$ (the reduced dynamic stiffness with frequency dependent matrices) is used as an estimate of the true modal frequencies.

Given the estimated modal frequencies ω_E, the terms of the viscous damping and stiffness matrices are finally approximated using

$$\left(C_E\right)_{ij} = \frac{\left(\mathrm{Im}\left(K_R(s)\right)\right)_{ij}}{mean\left(\left(\omega_E\right)_i, \left(\omega_E\right)_j\right)} \quad \text{and} \quad \left(K_E\right)_{ij} = \left(\mathrm{Re}\left(K_R(s)\right)\right)_{ij} - \delta_{ij} s^2 \qquad (18)$$

The third method is based on the fact that the dynamic flexibility $\left[\alpha(s)\right]$ is the quantity of interest for most predictions in structural dynamics. It thus seeks a good approximation of the complete flexibility matrix (inverse of the dynamic stiffness)

$$\left[M_V, C_V, K_V\right] = \arg\min \left\| \left[M_V s^2 + C_V s + K_V\right]^{-1} - \left[K_R(s)\right]^{-1} \right\|_2^2 \qquad (19)$$

This approximation can be obtained in two steps : determination of a partial fraction description (6) of a computed or measured dynamic flexibility and transformation to a complete approximation in the mass, constant viscous damping, constant stiffness form (as discussed in [9]).

When the constitutive laws have polynomial forms (model (2) for example) the partial fraction description can be determined using a complex eigenvalue solver. For more complex viscoelastic laws (fractional derivatives in particular [6]), a partial fraction description can be determined using identification methods as if the predictions were measurements [17]. The resulting model is only approximate but would typically be appropriate for all practical purposes.

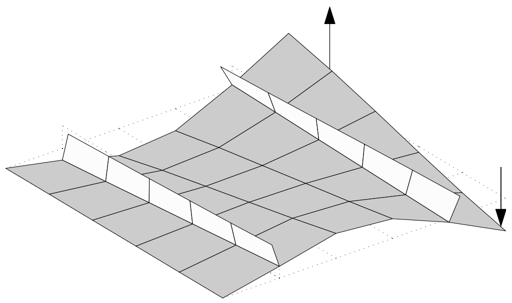

Fig. 7: 4th mode of the stiffened panel example with location of force input and displacement output. The skin modulus is assumed to have a standard viscoelastic behavior.

The validity of these three approaches will now be illustrated for the stiffened panel shown in figure 7. The panel is clamped on one edge and a metal/polymer/metal sandwich is used for the skin. To simplify the analysis, the skin is assumed to behave like a standard isotropic plate with a modulus of the 3 parameter form with the real zero at 400rd/s and the real pole at 500 rd/s. The associated amplitude and phase are shown in figure 8a where one sees the a peak loss factor (approximately the phase) of 0.1.

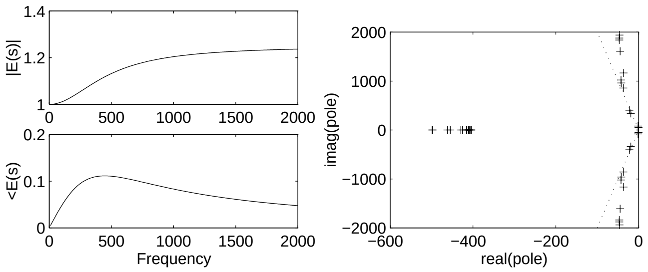

Fig. 8: **a)** amplitude and phase of the skin modulus **b)** position of poles in the complex plane. (····) 5% damping ratio or 10% loss factor line.

Figure 8b shows the location of panel poles below 2000 rd/s. One sees lightly damped poles whose damping ratio comes close to the 10% loss factor line near 500rd/s as expected from the modulus variation in figure 8a. The viscoelastic model also introduces as many real poles as DOFs linked to viscoelastic elements. The contribution of these poles is here negligible since the partial fraction model with their contribution truncated (dashed line of figure 9) has an amplitude difference much below 1/10 of the response and a very good phase match. One further notes that a complete approximation (19) of the model with the real pole truncated (dot dashed line in figure 9) is only marginally less accurate that the complex mode model.

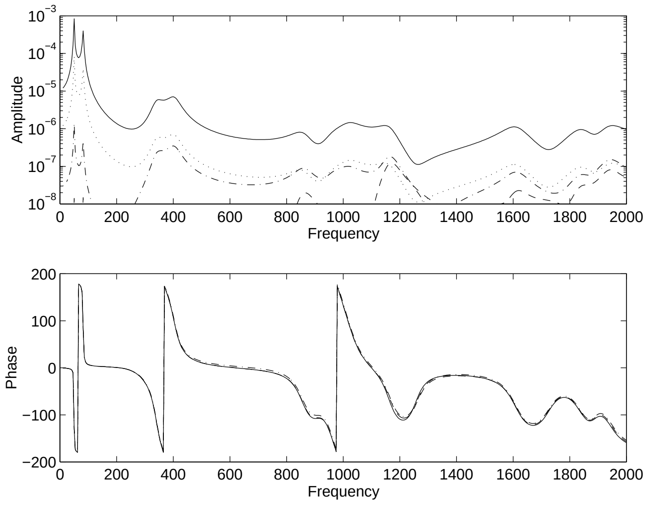

Fig. 9: Dynamic flexibility for force and displacement shown in figure 7. (—) exact H_{True} (---) $H_{\mathrm{True}} - H_{\mathrm{Reduced}}$ (real poles truncated) (·—·) $H_{\mathrm{True}} - H_{\mathrm{Viscous}}$ (mass, viscous damping, stiffness model), (····) $H_{\mathrm{True}}/10$.

One now considers a reduction basis containing the first 12 modes of the panel and the attachment mode linked to the considered input. These modes are computed for the asymptotic low frequency modulus (which is real). The initial basis is then orthonormalized with respect to the mass and stiffness so that the nominal reduced mass is the identity and the nominal reduced stiffness is diagonal.

The three approximation methods (17)-(19) were used to construct equivalent viscous models. The 1-1 and 5-5 terms of the associated stiffness $\left(\mathrm{Re}\left(K_R(s)\right)\right)_{ii} - m_{ii}s^2$ and damping $\mathrm{Im}\left(K_R(s)\right)$ contributions are compared in figure 10. The vertical dotted lines indicate the frequencies of the estimated lightly damped poles 1 and 5. The second stiffness approximation method (18) matches the dynamic stiffness exactly at this frequency. Its results are almost equal to those of the flexibility approximation method (19) which is expected since the estimated and true pole frequencies are slightly different. Method (17) gives an average match which will be shown to be inappropriate.

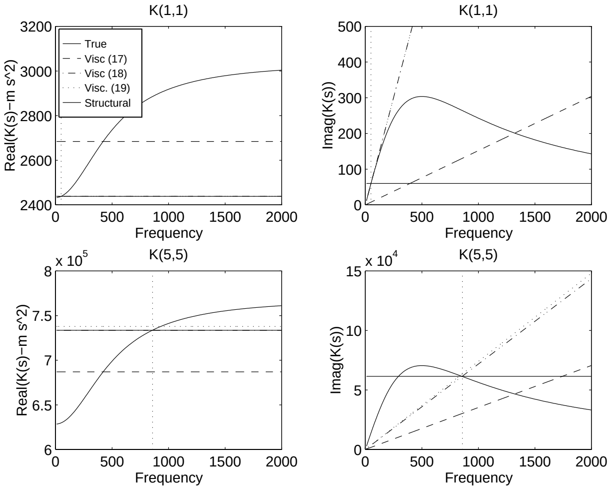

Fig. 10: Stiffness and damping contributions for the 1-1 and 5-5 terms of the reduced model.

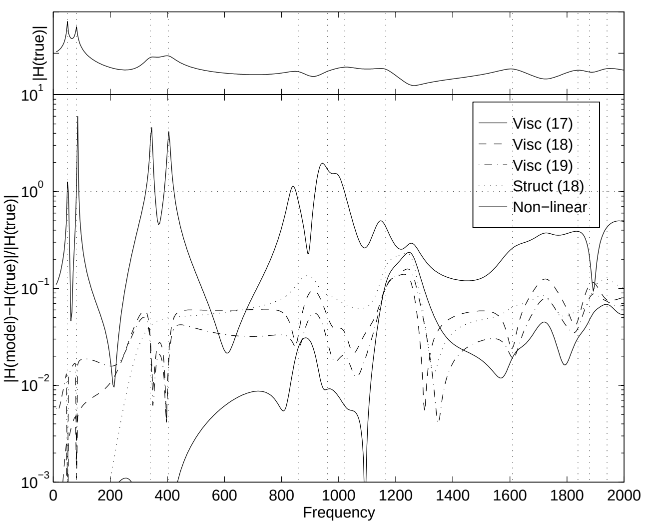

Fig. 11: For different equivalent models, quality of frequency response predictions measured by $\left|H_{Model} - H_{True}\right| / \left|H_{True}\right|$ which should be much lower than 1 (dotted line) at all frequencies.

Figure 11 indicates the quality of predicted transfer functions for the different considered models. The broadband method (17) is clearly inappropriate ($\left|H_{Model} - H_{True}\right| / \left|H_{True}\right|$ larger than 1 at many frequencies) while the other two approaches give results that are very good (the difference would be barely visible on overlaid FRFs).

Figure 11 finally indicates that the quality achieved using an equivalent viscous damping model would also be found if one constructed an equivalent structural damping model. In reduced model coordinates, the structural damping model does not however present a particular interest (see the interest in physical coordinates in the next section) and has the major drawback of only being meaningful in the frequency domain. Equivalent viscous damping models are thus preferable.

3.3. Going back to initial model coordinates

The existence of dynamically equivalent frequency independent reduced models was clearly established in section 3.2. The poor performance of method (17) clearly shows that individual terms of the equivalent mass, damping and stiffness properties must give a good approximation of the dynamic stiffness at very particular frequencies. The validity of method (18) is thus intimately linked to the use principal or quasi-principal coordinates for which characteristic frequencies are easily determined. Similarly method (19) uses complex modes to construct an equivalent model that is almost in principal coordinates [9].

Given an equivalent reduced model, one might however try to reconstruct associated matrices in the initial model coordinates. In particular, for a mass orthonormalized reduction basis ($T^T M T = I$) a full order matrix having the same reduced model is given by

$$\left[C_{Full}\right]_{N\times N} = \left[MT\right]_{N\times NR}\left[C_R\right]\left[T^T M\right]_{NR\times N} \tag{20}$$

Although typical finite element codes do not support this form, its numerical cost is quite limited if MT and C_R are stored and products $C_{Full}b$ are computed as three sequential products $\left(MT\left(C_R\left(T^T Mb\right)\right)\right)$.

For some applications, it might be interesting to find a set of local element properties such that the projection of the full order model is close the reduced one. Understanding limitations of this approach are important. Let us for example compare the panel of figure 7 assuming a 2% structural loss factor and a 1% viscous modal damping ratio. The associated frequency responses are given by

$$\begin{aligned} H_S &= \sum_{j=1}^{N} \frac{c\phi_j\phi_j^T b}{-\omega^2 + 2i\omega_j^2/100 + \omega_j^2} \\ H_V &= \sum_{j=1}^{N} \frac{c\phi_j\phi_j^T b}{-\omega^2 + 2i\omega\omega_j/100 + \omega_j^2} \end{aligned} \tag{21}$$

which are very close to each other since, at resonances ($\omega = \omega_j$), one has $\omega\omega_j = \omega_j^2$. From the orthogonality conditions of mass normalized modes, one can easily show that the viscous and structural damping matrices associated to a model of the form (21) are given by

$$[B] = 2i[K]/100 \text{ and } C = 2\left[M\phi\Omega\phi^T M\right]/100 \qquad (22)$$

Thus although the two models are dynamically equivalent, the structural damping matrix has the same sparsity pattern as the stiffness whereas the viscous damping model is generally full. Figure 12a even shows that the viscous damping matrix contains many important off diagonal terms connecting DOFs which are disconnected in the finite element model. To make the point even more convincing, the terms of C corresponding to zero terms in the K matrix were set to zero (enforce same sparsity pattern) and the resulting matrix was projected on the basis of the first 20 normal modes. Figure 12b clearly shows that the resulting modal damping matrix is far from being the nominal diagonal matrix $[\Gamma] = \left[\phi^T C \phi\right] = \left(2\left[{}^{\backslash}\Omega_{\backslash}\right]\right)/100$.

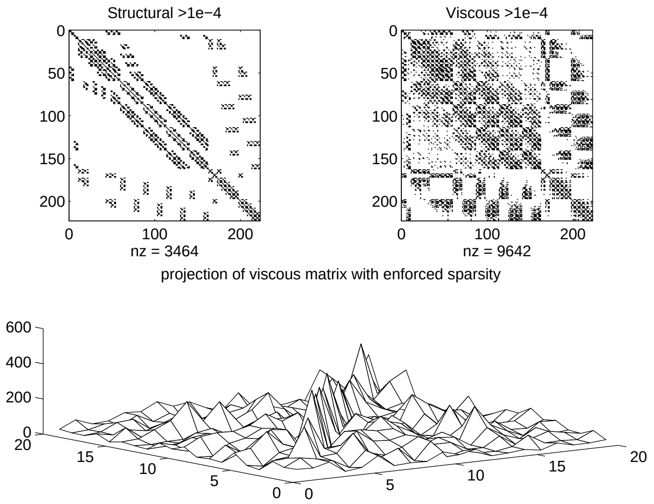

Fig. 12: **a)** Important off-diagonal terms ($C_{ij}^2 / C_{ii}C_{jj} < 1e-4$) of 1% damping structural and viscous damping matrices. **b)** Projection of matrix with enforced sparsity pattern.

4. CONCLUSIONS

For most applications in structural dynamics, the response is characterized by transfers which correspond to flexibilities (force to displacement transfer functions). Even for systems with significant frequency dependence of local elastic properties, dynamically equivalent second order models with frequency independent mass, stiffness and viscous or structural damping properties can be constructed provided that real poles have a negligible influence.

Analytically these equivalent reduced models can be efficiently used for viscoelastic predictions in the time or frequency domains. These models are however built using generalized degrees of freedom that are close to principal coordinates (i.e. associated to normal modes) and equivalent local damping properties cannot be found.

When determined experimentally, the information contained in equivalent models is not sufficient to characterize local frequency dependent damping properties without prior knowledge of the form of the actual damping mechanisms or careful tests of the local dynamic stiffness.

5. REFERENCES

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