An evaluation of modal testing results based on the force appropriation method.

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ABSTRACT

Modal parameters of structural systems can be obtained in many different ways. The force appropriation method uses one sine signal to generate forces at different points of a structure and adjusts the relative values of those forces so as to isolate a single mode. Such tests provide very accurate information on the modeshapes which is then complemented by specific tests to determine the mode damping and scaling (modal mass). This approach has been traditionally used for ground vibration testing of aircraft where the use of sine inputs is compatible with the need for large forces at very low frequencies.

After a presentation of the theory related to this testing methodology, this study applies, on the analytical model of actual aircraft, the tools generally used to extract experimental modal characteristics (frequency, damping, modeshape and modal mass) from test data. The knowledge of the true answer allows a real evaluation of the difficulties linked to different steps of the appropriation method. Issues addressed in particular are the definition of the actuator and sensor set-up, the determination of the appropriation forces rejecting unwanted modal contributions, and the accuracy of identified modal characteristics.

NOMENCLATURE

The paper uses the IMAC notation. Variables not contained in the standard are

b, c	input and output shape matrices
u, y	vector of input, outputs
$x \propto y$	x proportional to y
δ_{jk}	1 for $j=k$, 0 for $j \neq k$

I. INTRODUCTION

The force appropriation method (also called phase resonance testing or normal mode testing) has been traditionally used for ground vibration testing of aircraft [**Erreur! Source du renvoi introuvable.**].

The rationale for the appropriation method is that the application of an *ad hoc* generalized force (single signal applied at different actuators) allows the measurement of the response of a single normal mode. As other modes have small contributions, it is then possible to extract the modal characteristics of the considered mode (frequency ω_j, damping ratio ζ_j, modal mass m_{jIN}, modeshape ϕ) with great accuracy.

The different methods, used to appropriate a mode and determine its properties, are based on a number of assumptions which are not truly verified in practice. To improve the level of confidence in the quality of appropriation results, an effort was made to provide analytical simulations of the complete testing process. The resulting tools provide both a development environment for methods linked to particular steps of the appropriation method and a proofing tool for the *a posteriori* (possibly during the test) check of results.

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Section 2 details the theory behind the different steps leading to the identification of normal mode properties using the appropriation method. In section 3, an evaluation of limitations linked to these steps is done using the test simulation tools on a relatively complex modal model of true aircraft (see details in section 3.1). The use of an analytical model allows proper measures of errors linked to the different methods. The derivation of this analytical model from a real experiment allows the highlighting of a number of real world difficulties.

Problems addressed in section 3 include the definition of the actuator and sensor set-up, the determination of the appropriation forces rejecting unwanted modal contributions, and the accuracy of identified modal characteristics.

2. APPROPRIATION OF A NORMAL MODE

2.1. BASIC EQUATIONS

The theory of the force appropriation method is based on the initial assumption that there exist a viscously damped model of the system response of the form

$$\begin{aligned} \left[Ms^2 + Cs + K\right] \quad \{q\} &= b\{u(s)\} \\ \{\dot{y}(s)\} &= c \quad s\{q\} \end{aligned} \tag{1}$$

where it is assumed that

- the mass M, damping C, and stiffness K matrices are time-invariant, symmetric and positive-definite.
- the forces in the model coordinates depend linearly on forces (inputs) u in user defined coordinates $F_q = bu$. The matrix b is called the **input shape matrix.**
- the outputs y are linearly related to the model coordinates q through the sensor **output shape matrix** c ($y = cq$).

From model (1), normal modes are defined as solutions of the associated undamped eigenvalue problem

$$-M\phi\Omega + K\phi = 0 \tag{2}$$

where the normal modes ϕ verify two orthogonality conditions with respect to the mass and the stiffness

$$\phi^T M\phi = \mu \text{ and } \phi^T K\phi = \mu\Omega . \tag{3}$$

μ is a diagonal matrix of modal masses (which are non-physical quantities depending uniquely on the way the eigenvectors ϕ are scaled). In this paper, ϕ indicates the modeshapes associated with unity mass ($\mu = I$). For such mass normalized modeshapes, the generalized mass linked to an output c is given by

$$mg_{Cj} = 1 \Big/ \left(c\phi_j\right)^2 \tag{4}$$

Although modeshapes scaled using the constraint that $\mu = I$ contain a scaling information, the generalized mass at the sensor or excitation point with the maximum response will also be used, as done by most people, as a measure of the modeshape scaling.

Using the principal or modal coordinates $p = \lfloor\phi\rfloor^{-1} q$, a new representation of model (1) is found

$$\begin{aligned}\left[Is^2+\Gamma s+\Omega\right]\{p\}&=\phi^T bu\\ \dot{y}=c\phi \quad s\{p\}&\end{aligned} \tag{5}$$

where the mass is the unity matrix ($\mu = I$), the modal damping matrix $\Gamma = \phi^T C\phi$ is non-diagonal, and the modal stiffness matrix Ω (normal mode frequencies squared) is diagonal. $c\phi$ is the modal output shape matrix, and $\phi^T b$ the modal input shape matrix.

2.2. RATIONALE FOR THE FORCE APPROPRIATION METHOD

The extraction of parameters from a measured response would be relatively simple if the response only contained one mode. The appropriation methods thus tries to define a single generalized input, applied as a number of physical forces with the same time characteristics, which tends to excite the response of a single mode.

Mathematically, this objective can be stated as follows. For a set of actuators described by the modal input matrix $\lfloor \phi^T b \rfloor$, one seeks a constant and real (in general) vector u such that only one mode responds significantly ($\dot{p}_k \approx \delta_{jk}$).

If there were as many modes as actuators, one could impose an arbitrary response $\{\dot{p}\}$ of the different modes using the forces

$$u = \left[\phi^T b\right]^{-1}\left[Is + \Gamma + \Omega s^{-1}\right]\{\dot{p}\} \tag{6}$$

For example one could isolate mode j at its resonance ω_j (obtain a unit velocity response $\dot{p}_k = \delta_{jk}$) by applying the real forces

$$u_l = \sum_k \left[\phi^T b\right]_{lk}^{-1} \Gamma_{kj} \tag{7}$$

For a proportionally damped system (Γ diagonal), these forces would result in a simple one mode response of the form

$$\dot{y} = \frac{s\{c\phi_j\}\{b^T\phi_j\}^T}{s^2 + 2\zeta_j\omega_j s + \omega_j^2} u \tag{8}$$

Important characteristics of the single mode response (8) are shown in figure 1. For real inputs, the phase of all outputs transitions from ±90° to ∓90° with a passage at 0° at the resonance. Furthermore the real part of the response is only significant in a small band of width $2\zeta_j\omega_j$ centered at the resonance.

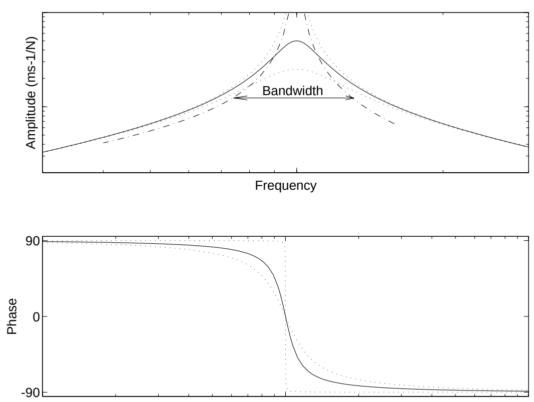

Figure 1: Velocity response of a single normal mode. (—) usual mode, (····) low and high damping envelope. (-·-) variation of bandwidth for increasing damping levels.

For a non-proportionally damped system (all physical systems), the forces (7) would only isolate the mode at the resonance. But the difference from the single mode response might not be significant.

2.3. APPROPRIATION IN PRACTICE

In reality, the modeshapes are not known and one cannot estimate the modal amplitudes p from the outputs y. Approximate criteria are thus used to evaluate the quality of mode isolation. Iterations are done using a phase criterion (to determine the resonance) and a quality criterion (to give a measure of the rejection of unwanted modes).

The **phase criterion** is based on the response of a single mode which, as shown in figure 1, is real at the resonance. One thus chooses a particular sensor or combination of sensors and defines an estimate of the resonance frequency, as the frequency for which the phase of a particular reference velocity sensor is real (imaginary for acceleration or displacement).

The phase criterion is a good approximation when other modes are not significantly coupled with the considered mode: when their bandwidth (see figure 1) is separated from the bandwidth of the considered mode. When one or a few modes are close to the considered mode, the phase criterion is only accurate when the forces injected already tend to isolate the considered mode.

The velocity response of an isolated mode at its resonance is purely real. The relative importance of the measured real and imaginary responses at the resonance (estimated with the phase criterion) thus provide an information on the quality of the appropriation (rejection of unwanted modes). This information is used in the form of different **quality criteria** such as those shown below

- The quality index used at ONERA gives a ratio of imaginary response to total response (its value is 1 for a perfect appropriation)

$$q(s) = 1 - \frac{\{|\mathrm{Im}\,\dot{y}|\}^T\{|\dot{y}|\}}{\{|\dot{y}|\}^T\{|\dot{y}|\}} \tag{9}$$

- The MMIF **[Erreur! Source du renvoi introuvable.]** gives a ratio of the quadrature energy (in-phase for displacement) to the total energy (its value is 0 for a perfect appropriation)

$$q(s) = \frac{\{|\mathrm{Im}\,\dot{y}|\}^T M\{|\mathrm{Im}\,\dot{y}|\}}{\{\dot{y}\}^H M\{\dot{y}\}} \tag{10}$$

- The inverse MMIF **[Erreur! Source du renvoi introuvable.]** maximizes ratio of the in-phase energy (quadrature for displacement) to the total energy (its value is 1 for a perfect appropriation)

$$q(s) = \frac{\{|\mathrm{Re}\,\dot{y}|\}^T M\{|\mathrm{Re}\,\dot{y}|\}}{\{\dot{y}\}^H M\{\dot{y}\}} \tag{11}$$

For the criteria (10) and (11), the mass matrix M is introduced to compare energies. In most cases, the mass is not known and the identity matrix is used instead (as for the criterion (9)). Except for sensor calibration issues, this has been found to be quite appropriate.

The appropriation method is usually performed through an empirical adjustment of inputs and resonance frequency so as to optimize the chosen quality criterion.

It was however noted by different authors that the mobility matrix Y gives a linear relationship between u and $\dot{y}$, $\dot{y}(s) = Y(s)u(s)$ so that the mode indicator functions (9)-(11) correspond to Rayleigh quotients of the form

$$q(s)=\frac{\{u\}^T A\{u\}}{\{u\}^T B\{u\}} \tag{12}$$

with $A=\left[\left|\mathrm{Im}\,\dot{Y}\right|\right]^T M\left[\left|\mathrm{Im}\,\dot{Y}\right|\right]$ and $B=\left[\dot{Y}\right]^H M\left[\dot{Y}\right]$ for the MMIF and $A=\left[\left|\mathrm{Re}\,\dot{Y}\right|\right]^T M\left[\left|\mathrm{Re}\,\dot{Y}\right|\right]$ and $B=\left[\dot{Y}\right]^H M\left[\dot{Y}\right]$ for the inverse MMIF. Note also the extended Asher method [**Erreur! Source du renvoi introuvable.**] where $A=\left[\left|\mathrm{Im}\,\dot{Y}\right|\right]^T\left[\left|\mathrm{Im}\,\dot{Y}\right|\right]$ and $B=I$.

Force inputs u that optimize q, thus correspond to the eigenvectors and eigenvalues (for q) solution of $Au=Buq$. When a transfer matrix is measured at the resonance, one can thus compute easily forces that optimize the quality criterion for the given measurement. Limitations linked to this approach will be addressed in section 3.3.

2.4. Parameter extraction from a single mode response

When properly found, appropriation forces allow to obtain measurements where the response is dominated by one mode. The work put into determining the appropriation forces is meant to allow an easier determination of the modal characteristics (frequency ω_j, damping ratio ζ_j, modal mass m_{jIN}, modeshape ϕ). The *complex power* (PC) and *force in quadrature* (FQ) methods, based on the assumption of a perfectly isolated mode with a response of the form (8), are classically used for this purpose.

From (8), the complex power input (transfer from force to collocated velocity) to the structure has the form

$$PC_{IN}=\frac{\dot{y}_{IN}}{u_{IN}}=\frac{s\{c_{IN}\phi_j\}\{c_{IN}\phi_j\}^T}{s^2+2\zeta_j\omega_j s+\omega_j^2}\stackrel{\Delta}{=}\frac{1}{m_{jIN}}\frac{s}{s^2+2\zeta_j\omega_j s+\omega_j^2} \tag{13}$$

which verifies four relations allowing the determination of modal characteristics

$$\begin{aligned}\omega_j &= \arg\max_{\omega_j} PC_{IN}\\ \partial PC_{IN}/\partial\omega\big|_{\omega_j} &= -1/2m_{jIN}\zeta_j^2\omega_j^2\\ PC_{IN}\big|_{\omega_j} &= 1/2m_{jIN}\zeta_j\omega_j\\ \phi &\propto \mathrm{Re}(\dot{y})\end{aligned} \tag{14}$$

If one performs a reinjection of a force in quadrature with the force applied at resonance $\tilde{u}_{IN}=(1+i\alpha)u_{IN}$. $=u_{IN}+i\alpha 2m_{jIN}\zeta_j\omega_j$ The response is then given by

$$\dot{y}_{IN}=\frac{su_{IN}}{m_{jIN}\left(s^2+2\zeta_j\omega_j s+\left(\omega_j^2+\alpha 2\zeta_j\omega_j\omega\right)\right)} \tag{15}$$

whose phase resonance is found for $\tilde{\omega}_j=\omega_j\sqrt{1-\zeta_j\alpha}$. The modal characteristics are thus determined by the relations

$$\begin{aligned}\omega_j &= \tilde{\omega}_j\big|_{\alpha=0}\\ \zeta_j &= \frac{1}{\omega_j}\frac{\partial\tilde{\omega}_j}{\partial\alpha}\\ m_{jIN} &= 1/2\zeta_j\omega_j\,\mathrm{Re}\left(PC_{IN}\right)\big|_{\omega_j}\\ \phi &\propto \mathrm{Re}(\dot{y})\end{aligned} \tag{16}$$

In practice, the mode is not perfectly isolated. Alternative approaches to the classical methods can thus introduce corrections for the effects of other modes. In section 3 for example the method called ML2 uses the following steps. Using a set of measured transfer functions near the resonance, the IDRC [**Erreur! Source du renvoi introuvable.**] identification method is used to determine the mode frequency and damping. The modal mass and shape are then determined using

$$\begin{aligned}m_{jIN} &= 1/2\zeta_j\omega_j\,\mathrm{Re}\left(PC_{IN}\right)\big|_{\omega_j}\\ \phi &\propto \mathrm{Re}(\dot{y})\end{aligned} \tag{17}$$

The usual objection to the validity of the appropriation approach is the fact that the number of actuators (and sensors) is lower than the number of modes so that formulas such as (7) are not applicable. For lightly damped systems however, the response of modes well separated from a given frequency is both small and almost imaginary (see figure 1). If the modal characteristics are based on measurements near the resonance of the mode of interest, the frequency, damping and mass estimates can take into account small residual terms and the modeshape estimate is insensitive to contributions to the imaginary response.

The adjustment of appropriation forces thus only needs to reject modes that are close in frequency (active in the considered bandwidth). As this is quite feasible (it basically amounts to the case with as many actuators as modes) the appropriation method has been widely used and very successful.

3. An evaluation of the appropriation approach

3.1. Model of the "Paris" aircraft

The Paris aircraft, designed in the 50s by Morane Saulnier, is a metallic four places subsonic jet motorized with 2 Turbomeca Marbore engines inside the 10 meter long fuselage. The aircraft has a T tail and a 10 meter span straight wing with two water tanks at the tips. The controls are classical. During the tests the elevator and rudder were clamped to the structure and the tanks full of water.

As part of an effort to join the two French Ground Vibration Test teams: ONERA and SOPEMEA, experiments were conducted on the Paris aircraft at the SOPEMEA plant. ONERA's part of the test [**Erreur! Source du renvoi introuvable.**] included the identification of a set of modes by the appropriation method (128 accelerometers and 14 shaker locations) and the acquisition of several transfer functions under different excitation conditions.

The test model retained for the rest of the present evaluation is composed of 9 normal modes (see table 1). It assumes linearity and proportional damping. Computations for this study were performed using the Structural Modeling Toolbox for MATLAB [**Erreur! Source du renvoi introuvable.**] and the ONERA Toolbox for the analysis of force appropriation results.

Table 1: Modes retained for the study.

#	Name	ω (Hz)	ζ (‰)	Nd#	mg (kgm^2)
1	aileron rotation	1.81	189.5	1010	12
2	2 node bending	4.77	9.4	1008	1330
3	tail roll	8.15	6.3	1025	33
4	anti. tank pitch	10.92	10.9	1000	105
5	sym. tank pitch	11.30	11.4	1004	77
6	tail yaw	11.34	15.7	110	43
7	3 node bending	15.15	17.9	109	101
8	sym. aileron rota.	16.00	19.7	998	9
9	harm. aileron rota.	17.06	15.2	998	23

For appropriation purposes, only a subset of *reference* sensors are used to evaluate the quality of appropriation with the criteria (9)-(11). The locations and directions of the sensors retained for the example are shown in figure 2. In all cases forces are applied at some of the reference sensor locations.

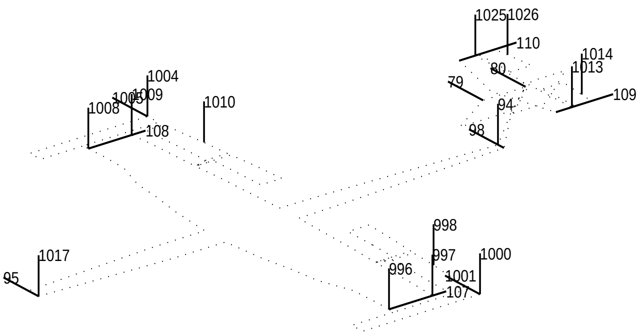

Figure 2: Locations and directions of reference sensors (actuators).

3.2. DEFINING A TEST SET-UP

The quality of an appropriation is measured by one of the criteria (9)-(11) which are computed based on sensor outputs. The value of these criteria mostly depend on the choice of retained sensors. In general, only those *reference* sensors are acquired during appropriation, since the full measurement is only needed at the appropriated resonance to determine the modeshape.

In the choice of reference sensors, the important factor is the geometrical independence of modeshapes. If the considered modeshapes are not geometrically different, the associated output shape matrix $c\phi$ is not well conditioned so that small differences in outputs y may correspond to large changes in the modal responses p (leading to inaccurate modeshape evaluations).

Initial Finite Element calculations, even when their frequencies and modeshapes are not very accurate, can be used for sensor placement. Simple checks of cross-MAC terms or more complex algorithms , such as the effective independence [**Erreur! Source du renvoi introuvable.**], lead to sensor placements preserving the geometrical independence. In the present case, no initial model was available so that the validity of the chosen set of reference sensors (shown in figure 2) could only be checked *a posteriori*.

Table 2: MAC (18) and singular values of $c\phi$ for an *a posteriori* check of reference sensor set validity.

#	1	2	3	4	5	6	7	8	9	*S.V.*
1	100	0	0	9	0	0	0	0	94	49.0
2	0	100	0	1	59	2	0	1	0	45.1
3	0	0	100	22	0	21	13	0	0	35.0
4	9	1	22	100	0	8	5	0	8	24.4
5	0	59	0	0	100	5	0	16	0	17.6
6	0	2	21	8	5	100	9	1	1	15.5
7	0	0	13	5	0	9	100	1	0	8.4
8	0	1	0	0	16	1	1	100	6	4.6
9	94	0	0	8	0	1	0	6	100	0.3

Table 2 clearly indicates that the chosen sensor set did not exactly respect the geometrical independence, particularly for modes 1/9 (two aileron rotation modes only distinguishable by the control column motion which was not measured) and 2/5 (where reference sensors on the wing mid-span would be needed). The similar modes are however well separated in frequency so that the sensor set did allow accurate measurements of the modes. (Modes 4-6 are the only ones with overlaying $2\zeta_j\omega_j$ bandwidths).

The phase criterion is based on the response of a particular sensor or group of sensors. For good results, this response must show a significant contribution of the mode of interest (if the response is too low, contributions of other modes may lead to significant shifts in frequency). Some *a priori* knowledge of modeshape is thus necessary to properly select the phase criterion sensor. In practice, this choice can be easily made from FRFs measured before the appropriation is actually performed (these FRFs also give an idea of the mode frequencies).

Practical considerations drive for the use of a minimal number of actuators to appropriate a given mode. Experience, the advance knowledge of general modeshape characteristics, and the rule of thumb that the force pattern should match the modeshape are generally sufficient to determine needed locations.

When the initial choice does not allow a good rejection of other modes, extra actuators are added in areas where the response is not in phase (thus allowing the rejection of modes with significant responses in those areas). For the determination of such areas, local quality criteria (based on wing, fuselage, tail, etc. sensors) are sometimes used.

As for sensors, the choice of actuators should preserve the geometrical independence of modes. One will however only try to achieve independence from close modes (by looking at the conditioning of the associated input shape matrix $\phi^T b$). Finally, the prediction of the modal amplitudes also provide a useful understanding of mode rejection.

For example, one could try to appropriate mode 5 (where the response is mostly localized to the tanks) using two actuators on the tanks (1000, 1004). Figure 3 shows the real and imaginary parts of the displacement responses of the 9 modes for forces optimizing the MMIF criterion (10). It clearly appears in this figure that mode 6 also has a significant contribution to the response and this contribution has a significant imaginary part so that modeshapes estimates would be inaccurate. (One obviously concludes that the number of actuators is insufficient).

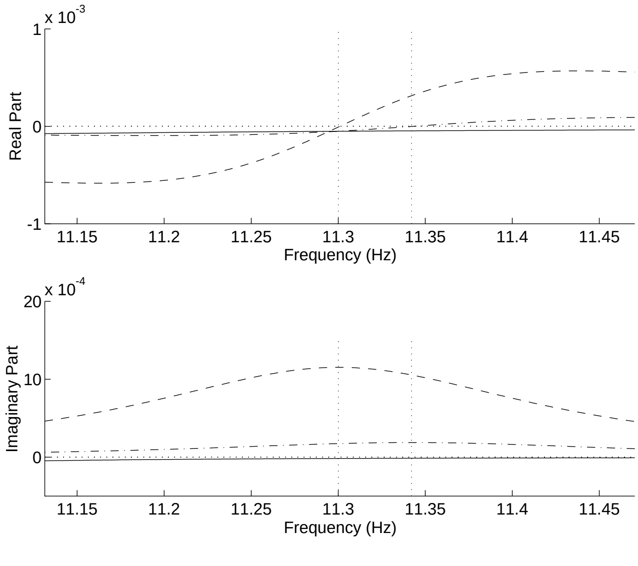

Figure 3: Modal displacements for 2 (1000, 1004) actuators and exact MMIF forces. (---) p_5, (-·-) p_6

3.3. "DIRECT" FORCE ESTIMATES BASED ON MEASURED FRF

When a transfer matrix between the force inputs and reference outputs is measured, "optimal" values of forces needed to obtain a good appropriation can be readily obtained through the optimization of the quotient (12) (eigensolution or any other method). This approach is subject to several problems.

- The determination of the resonance frequency is difficult.

The resonance is determined through the phase criterion or through the computation of the best “optimal” quality values over a range of measured frequencies. In both cases the accuracy of the frequency is limited because of noise in the measurement, approximate convergence, inappropriate sensor choices, or limitations of the frequency generators.

- The “optimal” forces can vary rapidly with frequency and be sensitive to noise in measured FRF.

For example, the exact MMIF (10) and associated normalized appropriation forces are shown in figure 4. For mode 5 at 11.3 Hz, the forces change rapidly near the resonance. For the analytical model, a standard deviation from the true resonance of 0.002 Hz leads to variations of the estimated forces with a standard deviation above 80 % of their nominal value.

Similarly the predicted forces can be very sensitive to noise in the measured data. For 5% random noise on the FRF, the standard deviations on the predicted forces are low for mode 5 (less than 8% for significant forces) but high for mode 6 at 11.34 Hz (above 25 % for all four considered inputs, see also results in table 5). Note that this is not the result that would be expected from figure 4, where MMIF forces vary rapidly near mode 5 and slowly near mode 6.

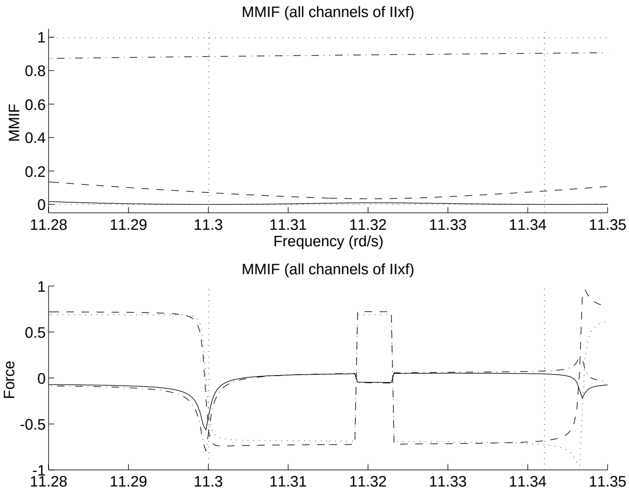

Figure 4: MMIF appropriation forces for 4 (1000, 1004, 109, 110) actuators. The vertical dotted lines indicate the frequencies of the poles.

(Note that the variations considered here happen without further difficulties such as a switch in optimal forces considered in Ref. [**Erreur! Source du renvoi introuvable.**]).

- The use of the transfer function matrix is based on the assumption of linearity.

In practice, appropriation is used to test non-linear systems (such as airplanes) where the modal characteristics change for each form and level of input (each row/column of the transfer matrix).

The transfer matrix from excitation on the wing tips (1000 and 1004) and the tail (109 and 110) to the reference sensors were measured on the *Paris* using sine excitation. The MMIF values shown in figure 5 seem to indicate the presence of two modes (with frequencies indicated as vertical lines). A more thorough check of the data, however, shows that there is only one mode whose frequency is 10.9 Hz for excitation by the wings and 11.0 Hz for excitation by the tail. The shift in resonance frequency is simply confirmed by plotting cross-transfers which would overlay exactly for a linear reciprocal system.

Note that the level of response in the transfers has also changed which implies a modification of both the mode frequency and shape. A reciprocity check for a non-linear system should be performed at equal amplitude levels for the non-linearity. Here location of the non-linearity is not known and the response of known points was not maintained constant (a factor 2 for point 109). Therefore, the results shown in the figure weren’t proper to test for reciprocity.

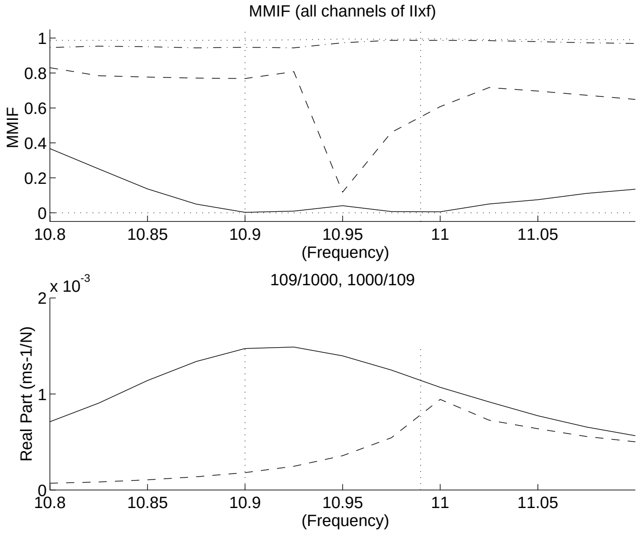

Figure 5: MMIF values and the cross transfers (1000 to 109 and 109 to 1000), for a FRF matrix obtained through a sine test. This data seems to indicate the presence of two modes when there is only one.

It is unclear that one can reach with different excitations a single operating point, where the modal characteristics would be sufficiently constant (the sensitivity of the modal frequency found here is relatively small for an aircraft mode), It has been proposed [**Erreur! Source du renvoi introuvable.**] to construct the FRF matrix using small amplitude/phase variations from the currently estimated appropriation force, but demonstrations of this approach for complex systems have not been shown.

3.4. EXTRACTION OF MODAL CHARACTERISTICS

Passing the difficulties linked to the optimization of the quality criterion, the validity of the method should be judged by the accuracy of predicted modal characteristics. Frequencies and damping can be compared directly.

Modal masses and modeshapes are interlinked quantities so that appropriate criteria must be defined. The Modal Assurance Criterion

$$MAC_{jk} = \frac{\sum_l c_l \phi_j c_l \phi_k}{\sqrt{\sum_l \left(c_l \phi_j\right)^2 \sum_l \left(c_l \phi_k\right)^2}} \qquad (18)$$

gives an indication of the correlation between two shapes, without reference to scaling (modal masses must be compared) and without guarantee that the shapes are equal (correlated ≠ equal). The relative modeshape error

$$\left\| c\phi_j - c\phi_j \right\| / \left\| c\phi_j \right\| \qquad (19)$$

implies the use of mass normalized modeshapes and is a much less forgiving criterion (for 0 relative modeshape error the two modeshapes are equal at all measured sensor locations and the modal masses are the same).

Table 3 summarizes the results of an analysis of the final result qualities for optimal MMIF forces at true resonance frequency combined with the FQ method (16) for modal mass and damping extraction. Except for mode 5, all the results are very accurate.

Table 3: Summary of final result accuracy for optimal MMIF forces at true resonance frequency combined with the FQ method (16) for modal mass and damping extraction. Mode number # and frequency ω, number of actuator positions *na*, MMIF quality criterion *q*, error on predicted frequency $\Delta\omega$ and damping ratio $\Delta\zeta$, modal assurance criterion MAC (18) with error on modal mass at two different nodes Δmg_i, relative error on mass normalized modeshape (19).

#	ω	*na q*	$\Delta\omega$ %	$\Delta\zeta$ %	MAC	Δm_1%	Δm_2%	$\Delta\phi/\phi$ (%)
1	1.81	20.00	0.0	0.0	1.00	0.0	0.0	0.0
2	4.77	20.00	0.0	0.0	1.00	0.9	0.9	0.5
3	8.15	20.00	0.0	0.7	1.00	0.8	0.7	0.4
4	10.92	40.01	0.0	0.7	1.00	0.6	-20.0	0.5
5	11.30	20.01	0.0	2.8	0.95	0.6	-15.7	24.7
6	11.34	40.00	0.0	0.0	1.00	0.1	0.5	1.0
7	15.15	30.02	-0.4	2.7	1.00	-0.4	-0.4	0.7
8	16.00	20.00	0.0	0.3	1.00	0.5	-0.5	0.3
9	17.06	20.01	-0.3	1.3	1.00	-1.0	-1.3	0.5

For mode 5, the damping is well predicted and the first modal mass also. The modeshape is however not correct as indicated by the high relative error (25%) and to some extent the MAC (although most people think that .95 is good). Figure 6 shows a comparison of the modeshape and its estimate. It clearly appears that the main error is located on the tail. This corresponds to the result shown in figure 4, where the modal amplitude of mode 6 (tail yaw) is significant.

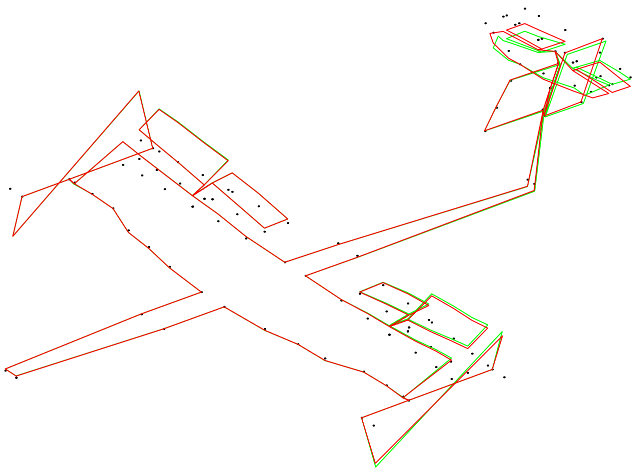

Figure 6: True and identified modeshapes (mode 5).

The extremely good quality criterion would not allow a detection of the error made. *A priori* analyses using analytical modeshape predictions or *a posteriori* evaluations such as the analyses made here should thus be performed to guarantee the validity of the results.

To understand the reason for the error on mode 5, the appropriation was performed on the model using 1, 2 or 3 actuators. The results shown in table 4, clearly indicate that 3 actuators are needed to appropriate mode 5. This corresponds to the number of close modes (i.e. 4-6) which confirms the rule of thumb "as many actuators as close modes".

Table 4: Errors in % in frequency, damping and generalized mass prediction for mode 5 at 11.3 Hz using MMIF appropriation forces and 1 (1000), 2 (1000, 1004), and 3 (1000, 1004, 109) actuator positions.

#	Exact	1 act. FQ	1 act. ML2	2 act. FQ	2 act. ML2	3 act. FQ	3 act. ML2
MMIF		0.104		0.004		0.000	
ω (Hz)	11.30	-0.6	0.2	-0.0	0.0	0.0	0.0
ζ (‰)	12.21	42.0	13.7	1.3	1.9	0.0	0.0
mg (1004)	77.38	73.4	-11.6	1.7	4.9	0.1	0.0
mg (1000)	131.51	-53.4	-32.0	-15.4	-22.2	-1.0	-1.0

Table 4 also gives a comparison of results obtained using MMIF appropriation forces at the true resonance but the FQ and ML2 methods for parameter estimation.

From the results shown and other evaluations, it appears that estimates based on the ML2 approach are more accurate for very bad appropriations (such as the case with 1 actuator), but give sensibly equivalent results otherwise (with equal chances of being slightly wrong). The *FQ* and *PC* methods are thus sufficient for parameter extraction of an appropriated mode from data near its resonance. Advanced identification methods, such as IDRC used in ML2, should be used for parameter extraction in non-appropriated cases.

Finally, an evaluation of the integrated methodology was performed. For this evaluation, 5% normal noise was added on the real and imaginary parts of FRFs and the MMIF appropriation forces were computed at the phase resonance (rather than the true mode resonance). (As for the results of table 3, the *FQ* (16) method was used to extract modal parameters from predicted FRF near the resonance). Table 5 summarizes results for one particular evaluation.

The results shown in table 5 and the fact that they are only slightly worse than those shown in table 3 give a final demonstration of the robustness of the appropriation method. For modes 1 2 3 7 9, the relative scaled modeshape error is close to the 5% lower limit imposed by the added noise. Modes 3 and 6 are well identified even though the applied forces differ significantly from those needed for an optimal appropriation in the noise-free case.

Table 5: Summary of final result accuracy for sub-optimal MMIF forces at phase resonance with 5% normal noise on real and imaginary parts of all predicted FRFs. (see table 3 except for Δu which represents the relative error on the input vector *u*)

#	Δu %	*na q*	$\Delta\omega$ %	$\Delta\zeta$ %	MAC	Δm_1%	Δm_2%	$\Delta\phi/\phi$ (%)
1	3	20.00	0.1	-14.3	1.00	8.0	31.1	6.7
2	15	20.00	-0.1	2.4	1.00	-11.3	8.9	4.0
3	92	20.00	0.0	0.4	1.00	4.1	-5.6	5.1
4	2	40.01	-0.1	-8.0	1.00	9.5	4.3	14.9
5	2	20.01	0.0	-0.6	0.93	-1.0	-8.2	36.7
6	132	40.00	0.0	1.7	0.99	0.9	-4.2	9.6
7	1	30.02	-0.5	-0.2	0.99	6.4	-1.2	7.3
8	8	20.00	0.0	-1.1	0.99	6.9	-3.9	10.6
9	1	20.01	-0.2	2.3	1.00	2.6	15.6	3.2

4. CONCLUSIONS

Normal mode testing is a very efficient testing approach but it involves a number of choices which need to be well understood. The present study showed how most important issues could be addressed through the use of testing methods on an analytical model derived from test of an actual aircraft. Main points highlighted in this study were

- the need to properly choose actuators and reference sensors
- the difficulties linked to the use of force estimates based on measured FRFs (for non-linear systems in particular)
- the accuracy of the traditional single mode parameter identification methods (for appropriated modes)
- the robustness of the overall parameter identification process to noise and resonance estimation with the phase criterion.

By including models of more complex phenomena such as non-proportional damping and non-linearities, the tools developed for this study will allow better proofing of test results and evaluation of new techniques for appropriation (in particular for the reduction of the time needed to identify a mode).

5. References

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